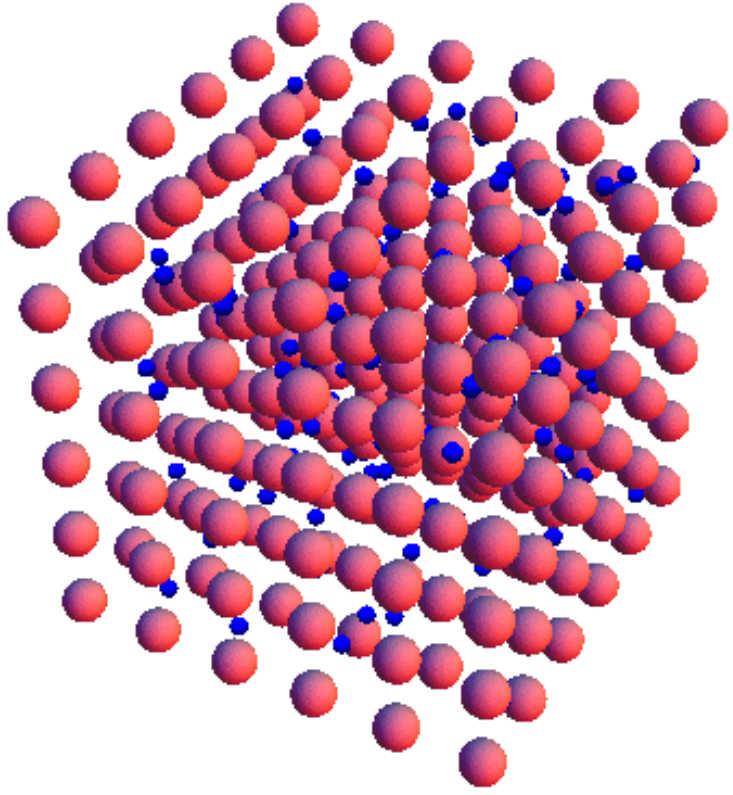
Capítulo 5: Modelo de electrones libres para metales



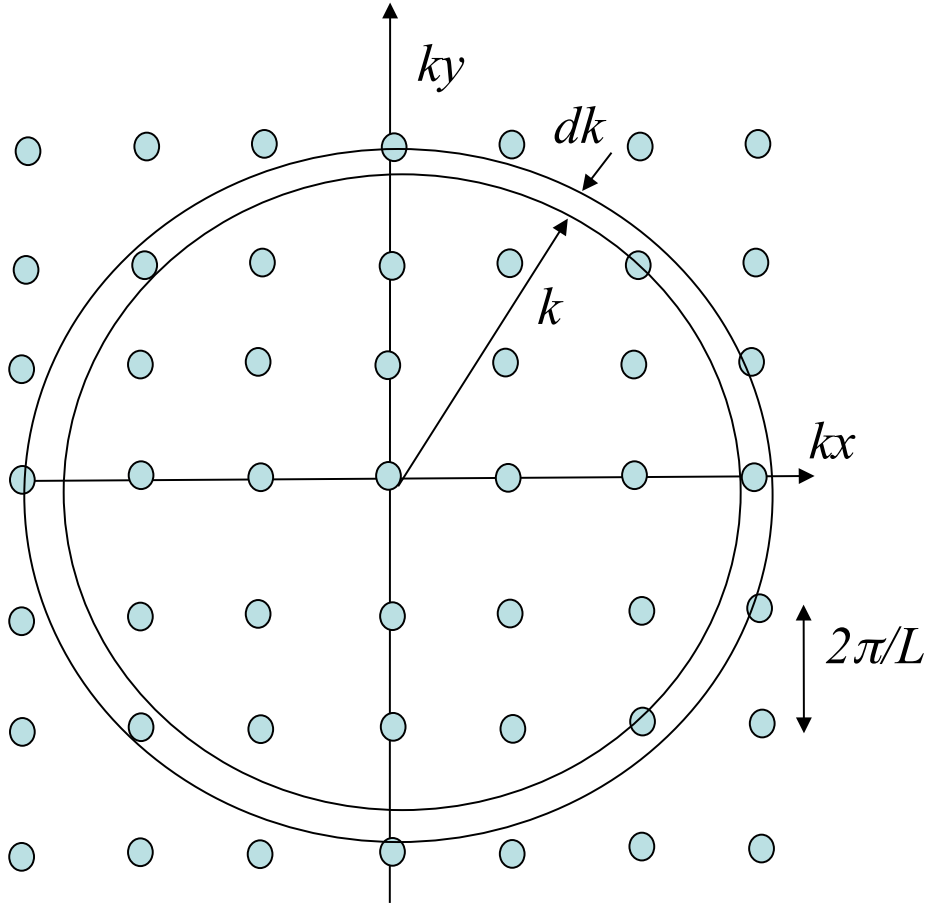


Table 3 Crystal structures of the elements

The data given are at room temperature for the most common form, or at the stated temperature in deg K. For further descriptions of the elements see Wyckoff, Vol. 1, Chap. 2. Structures labeled complex are described there.

vdW

Cov. + vdW

Covalente

unión metálica

Crystal structure.
a lattice parameter, in Å
c lattice parameter, in Å.

H[1] 4K hcp 3.75 6.12																	He[4] 2K hcp 3.57 5.83
Li 78K bcc 3.491	Be hcp 2.27 3.59											B rhomb.	C diamond 3.567	N 20K cubic 5.66 (N_2)	O complex (O_2)	F	Ne 4K fcc 4.46
Na 5K bcc 4.225	Mg hcp 3.21 5.21											Al fcc 4.05	Si diamond 5.430	P complex	S complex	Cl complex (Cl_2)	Ar 4K fcc 5.31
K 5K bcc 5.225	Ca fcc 5.58	Sc hcp 3.31 5.27	Ti hcp 2.95 4.68	V bcc 3.03	Cr bcc 2.88	Mn cubic complex	Fe bcc 2.87	Co hcp 2.51 4.07	Ni fcc 3.52	Cu fcc 3.61	Zn hcp 2.66 4.95	Ga complex	Ge diamond 5.658	As rhomb.	Se hex. chains	Br complex (Br_2)	Kr 4K fcc 5.64
Rb 5K bcc 5.585	Sr fcc 6.08	Y hcp 3.65 5.73	Zr hcp 3.23 5.15	Nb bcc 3.30	Mo bcc 3.15	Tc hcp 2.74 4.40	Ru hcp 2.71 4.28	Rh fcc 3.80	Pd fcc 3.89	Ag fcc 4.09	Cd hcp 2.98 5.62	In tetr. 3.25 4.95	Sn (α) diamond 6.49	Sb rhomb.	Te hex. chains	I complex (I_2)	Xe 4K fcc 6.13
Cs 5K bcc 6.045	Ba bcc 5.02	La hex. 3.77 *ABAC*	Hf hcp 3.19 5.05	Ta bcc 3.30	W bcc 3.16	Re hcp 2.76 4.46	Os hcp 2.74 4.32	Ir fcc 3.84	Pt fcc 3.92	Au fcc 4.08	Hg rhomb.	Tl hcp 3.46 5.52	Pb fcc 4.95	Bi rhomb.	Po sc 3.34	At —	Rn —
Fr —	Ra —	Ac fcc 5.31															

Ce fcc 5.16	Pr hex. 3.67 *ABAC*	Nd hex. 3.66	Pm —	Sm complex	Eu bcc 4.58	Gd hcp 3.63 5.78	Tb hcp 3.60 5.70	Dy hcp 3.59 5.65	Ho hcp 3.58 5.62	Er hcp 3.56 5.59	Tm hcp 3.54 5.56	Yb fcc 5.48	Lu hcp 3.50 5.55
Th fcc 5.08	Pa tetr. 3.92 3.24	U complex	Np complex	Pu complex	Am hex. 3.64 *ABAC*	Cm —	Bk —	Cf —	Es —	Fm —	Md —	No —	Lr —

Metales

electrones que se mueven libremente a través del material ⟹ electrones de conducción

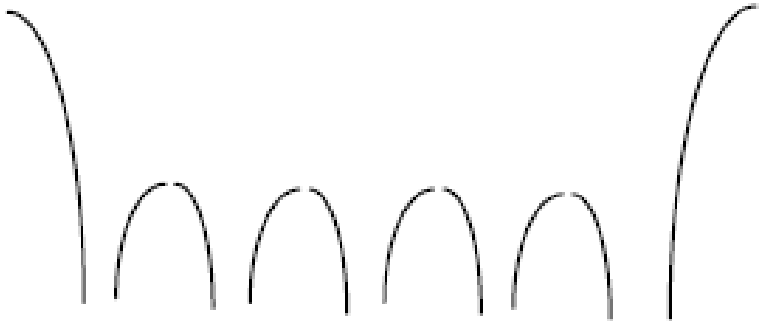
Modelo : Gas de electrones libres

Drude (1900)

Considero un gas clásico para estudiar conductividad

Sommerfeld (1927)

Aplicó la estadística de Fermi-Dirac

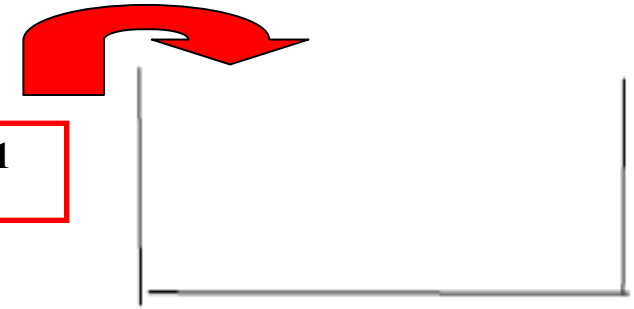


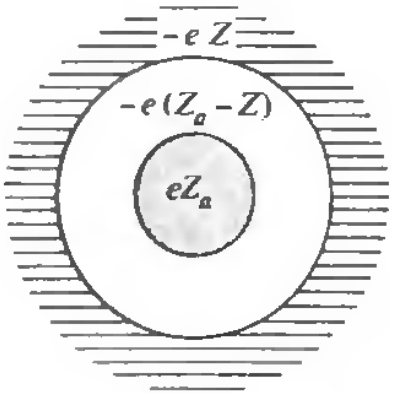
Potencial cristalino

Potencial del modelo

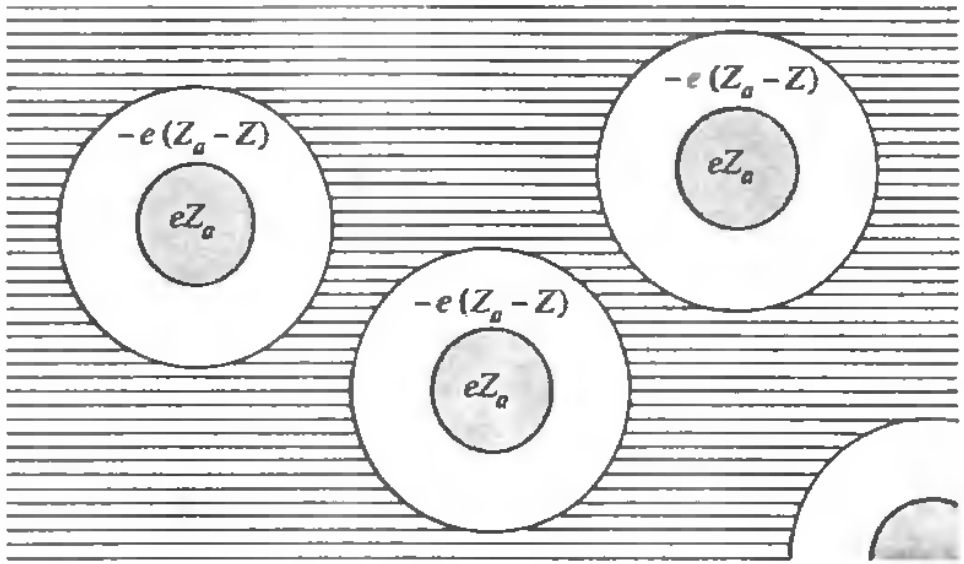
Sodio metálico

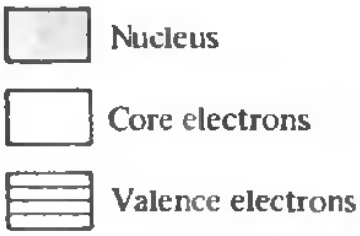
Na: $1s^2$ $2s^2$ $2p^6$ $3s^1$

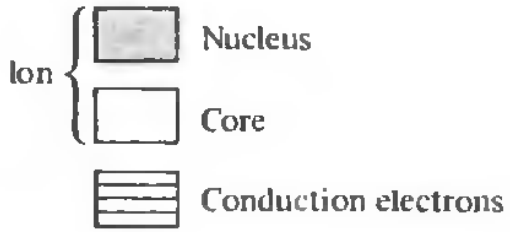




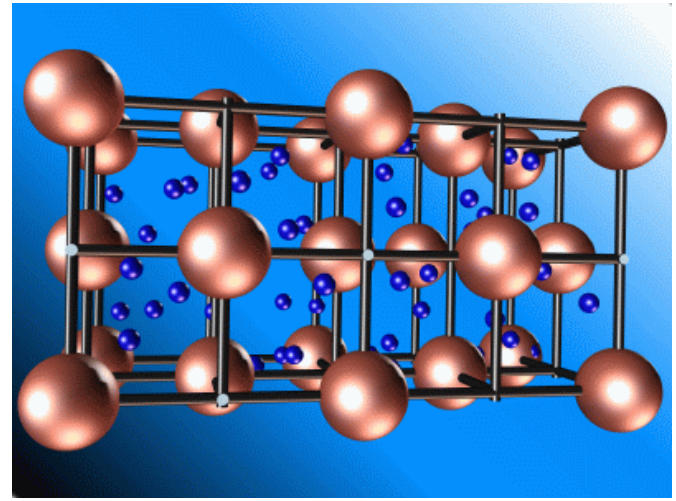
Model potential







Densidad electrónica (electrones/cm^3) ?



$$n = 0.6022 \times 10^{24} \frac{Z\rho_m}{A}.$$

Z: valencia

A: masa atómica (g/mol)

ρ_m = densidad (g/cm^3)

Otra forma de expresar la densidad:

$$\frac{V}{N} = \frac{1}{n} = \frac{4\pi r_s{}^3}{3}; \quad r_s = \left(\frac{3}{4\pi n}\right)^{1/3}.$$

ELEMENT	Z	$n\ (10^{22}/\mathrm{cm}^3)$	r_s(Å)	r_s/a_0
Li (78 K)	1	4.70	1.72	3.25
Na (5 K)	1	2.65	2.08	3.93
K (5 K)	1	1.40	2.57	4.86
Rb (5 K)	1	1.15	2.75	5.20
Cs (5 K)	1	0.91	2.98	5.62
Cu	1	8.47	1.41	2.67
Ag	1	5.86	1.60	3.02
Au	1	5.90	1.59	3.01
Be	2	24.7	0.99	1.87
Mg	2	8.61	1.41	2.66
Ca	2	4.61	1.73	3.27
Sr	2	3.55	1.89	3.57
Ba	2	3.15	1.96	3.71
Nb	1	5.56	1.63	3.07
Fe	2	17.0	1.12	2.12
Mn (α)	2	16.5	1.13	2.14
Zn	2	13.2	1.22	2.30
Cd	2	9.27	1.37	2.59
Hg (78 K)	2	8.65	1.40	2.65
Al	3	18.1	1.10	2.07
Ga	3	15.4	1.16	2.19
In	3	11.5	1.27	2.41
Tl	3	10.5	1.31	2.48
Sn	4	14.8	1.17	2.22
Pb	4	13.2	1.22	2.30
Bi	5	14.1	1.19	2.25
Sb	5	16.5	1.13	2.14

Modelo de Sommerfeld

Para un gas de N_e electrones :

$$\left(-\frac{\hbar^2}{2m_e} \sum_{i=1}^{N_e} \nabla_i^2 \right) \Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_{N_e}) = E\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_{N_e})$$

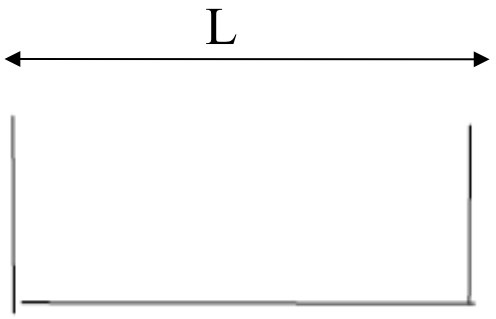
Si despreciamos la interacción e-e, resolvemos el problema de 1 e^-

$$-\frac{\hbar^2}{2m_e} \nabla_i^2 \psi(\boldsymbol{r}_i) \equiv -\frac{\hbar^2}{2m_e} \left(\frac{\partial^2}{\partial x_i^2} + \frac{\partial^2}{\partial y_i^2} + \frac{\partial^2}{\partial z_i^2} \right) \psi(\boldsymbol{r}_i) = \varepsilon \psi(\boldsymbol{r}_i)$$

y llenamos los niveles de energía con los N_e electrones

Ej.: 1D

$$\mathcal{H}\psi_n = -\frac{\hbar^2}{2m}\frac{d^2\psi_n}{dx^2} = \epsilon_n\psi_n$$



Model potential

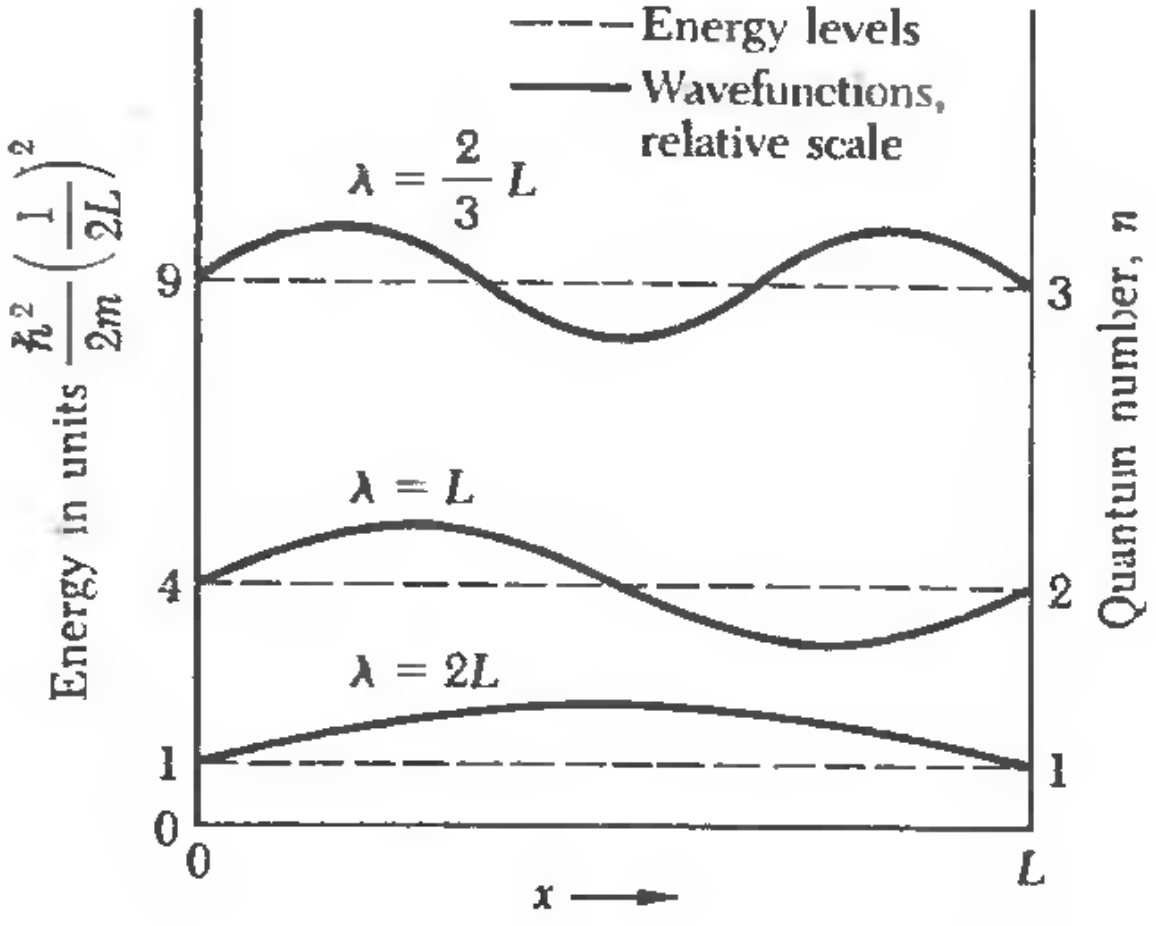
Condiciones de contorno $\Longrightarrow$ $\psi_n(0) = 0;\ \psi_n(L) = 0$

$$\psi_n = A \sin\left(\frac{2\pi}{\lambda_n}x\right)$$

$$\frac{2\pi}{\lambda_n} = k = n\,\pi / L \qquad n=1,2,\ldots$$

$$\frac{d\psi_n}{dx} = A\left(\frac{n\pi}{L}\right)\cos\left(\frac{n\pi}{L}x\right)$$

$$\frac{d^2\psi_n}{dx^2} = -A\left(\frac{n\pi}{L}\right)^2 \sin\left(\frac{n\pi}{L}x\right) \Longrightarrow \epsilon_n = \frac{\hbar^2}{2m}\left(\frac{n\pi}{L}\right)^2$$



Números cuánticos: n = 1, 2, 3, 4,….

s = + ½ , - ½ (↑ , ↓)

Lleno los niveles con los N_e electrones

Ej.: $N_e = 6$

n	m_s	Electron occupancy	n	m_s	Electron occupancy
1	↑	1	3	↑	1
1	↓	1	3	↓	1
2	↑	1	4	↑	0
2	↓	1	4	↓	0

n_F: n correspondiente al último nivel ocupado **$n_F = 3$**

En general, si N es par se tiene: $n_F = N/2$

en un cristal $N \approx 10^{23}$

Definimos la Energía de Fermi como la energía del último nivel ocupado

$$\epsilon_F = \frac{\hbar^2}{2m}\left(\frac{n_F \pi}{L}\right)^2 = \frac{\hbar^2}{2m}\left(\frac{N\pi}{2L}\right)^2$$

La energía de Fermi es función de N/L (densidad electrónica)

Para el gas en 3D

$$\psi(x,y,z) = \sin\frac{\pi n_x x}{L_x}\ \sin\frac{\pi n_y y}{L_y}\ \sin\frac{\pi n_z z}{L_z}$$

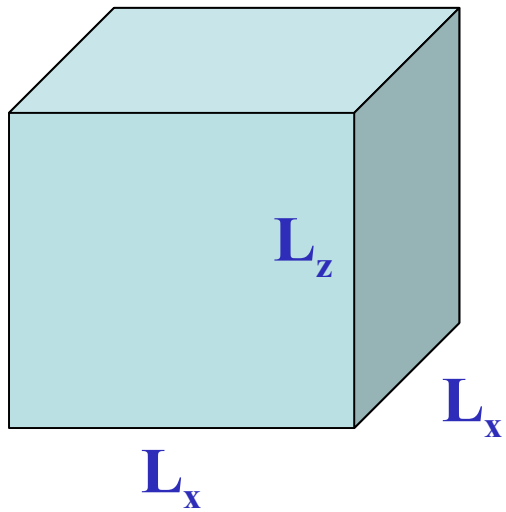
$$\varepsilon = \frac{\hbar^2\pi^2}{2m_e}\left[\left(\frac{n_x}{L_x}\right)^2 + \left(\frac{n_y}{L_y}\right)^2 + \left(\frac{n_z}{L_z}\right)^2\right] \qquad n_i=1,2,3,..$$

$$\varepsilon_F = \frac{\hbar^2}{2m_e}(3\pi^2 n_e)^{2/3}$$

Estas condiciones de contorno no son adecuadas para estudiar las propiedades de bulk de un material

Condiciones periódicas de contorno (Born-von Karman)

$$-\frac{\hbar^2}{2m_e}\nabla_i^2\psi(\boldsymbol{r}_i) \equiv -\frac{\hbar^2}{2m_e}\left(\frac{\partial^2}{\partial x_i^2}+\frac{\partial^2}{\partial y_i^2}+\frac{\partial^2}{\partial z_i^2}\right)\psi(\boldsymbol{r}_i) = \varepsilon\psi(\boldsymbol{r}_i)$$



$$\psi(x, y, z + L) = \psi(x, y, z),$$
$$\psi(x, y + L, z) = \psi(x, y, z),$$
$$\psi(x + L, y, z) = \psi(x, y, z).$$

Las soluciones son: $\psi_{\boldsymbol{k}}(\boldsymbol{r}) = \frac{1}{\sqrt{V}}e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$

$$\varepsilon_{\boldsymbol{k}} = \frac{\hbar^2 k^2}{2m_e} = \frac{\hbar^2}{2m_e}(k_x^2 + k_y^2 + k_z^2)$$

★ **Las funciones de onda** $\psi_{\boldsymbol{k}}(r) = \frac{1}{\sqrt{V}} \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}$ **son autofunciones del operador momento**

$$p = \frac{\hbar}{\mathrm{i}} \frac{\partial}{\partial r} = \frac{\hbar}{\mathrm{i}} \nabla$$

Poseen momento lineal definido $p = \hbar k.$

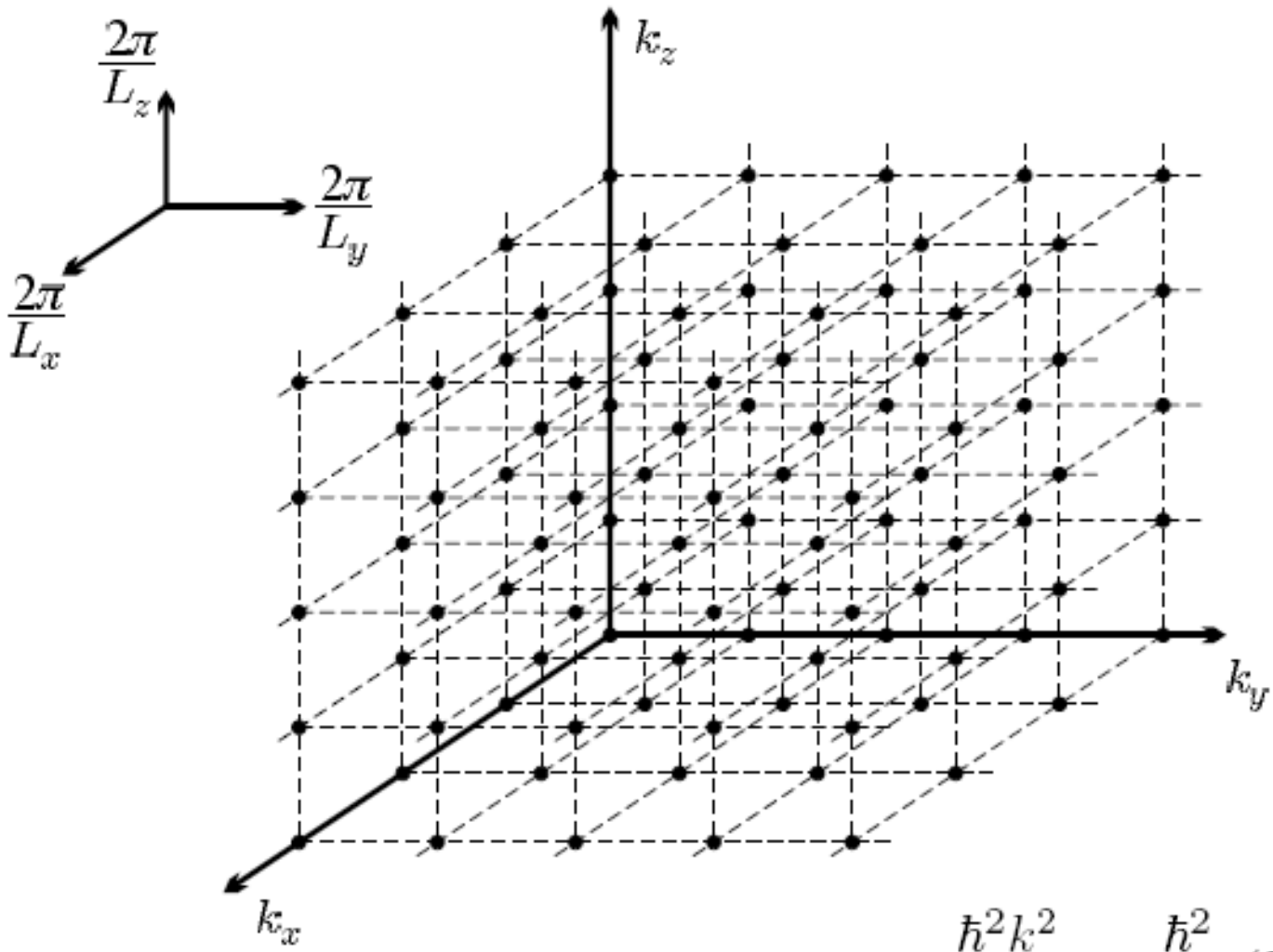
Valores posibles de $\vec{k}$? $\psi(x + L, y, z) = \psi(x, y, z)$

$$\mathrm{e}^{\mathrm{i}k_x L_x} = \mathrm{e}^{\mathrm{i}k_y L_y} = \mathrm{e}^{\mathrm{i}k_z L_z} = 1$$

$$k_x = \frac{2\pi}{L_x} n_x\,, \qquad k_y = \frac{2\pi}{L_y} n_y\,, \qquad k_z = \frac{2\pi}{L_z} n_z$$

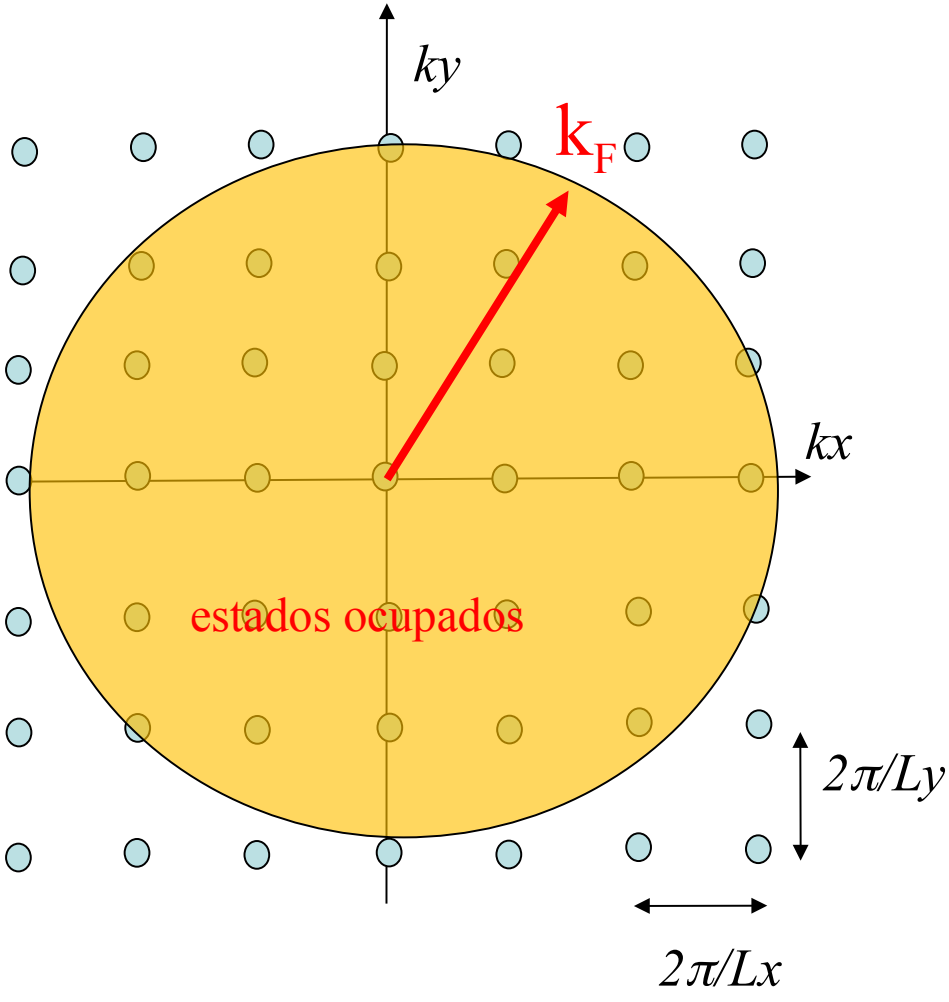
n_i entero

$$\boldsymbol{k} = \frac{2\pi n_x}{L_x}\hat{\boldsymbol{x}} + \frac{2\pi n_y}{L_y}\hat{\boldsymbol{y}} + \frac{2\pi n_z}{L_z}\hat{\boldsymbol{z}}$$



$$\varepsilon_{\boldsymbol{k}} = \frac{\hbar^2 k^2}{2m_e} = \frac{\hbar^2}{2m_e}(k_x^2 + k_y^2 + k_z^2)$$

Para N_e electrones:



$$\varepsilon_F = \frac{\hbar^2 k_F^2}{2m_e}$$

Energía de Fermi

$$v_F = \frac{\hbar k_F}{m_e}$$

Velocidad de Fermi

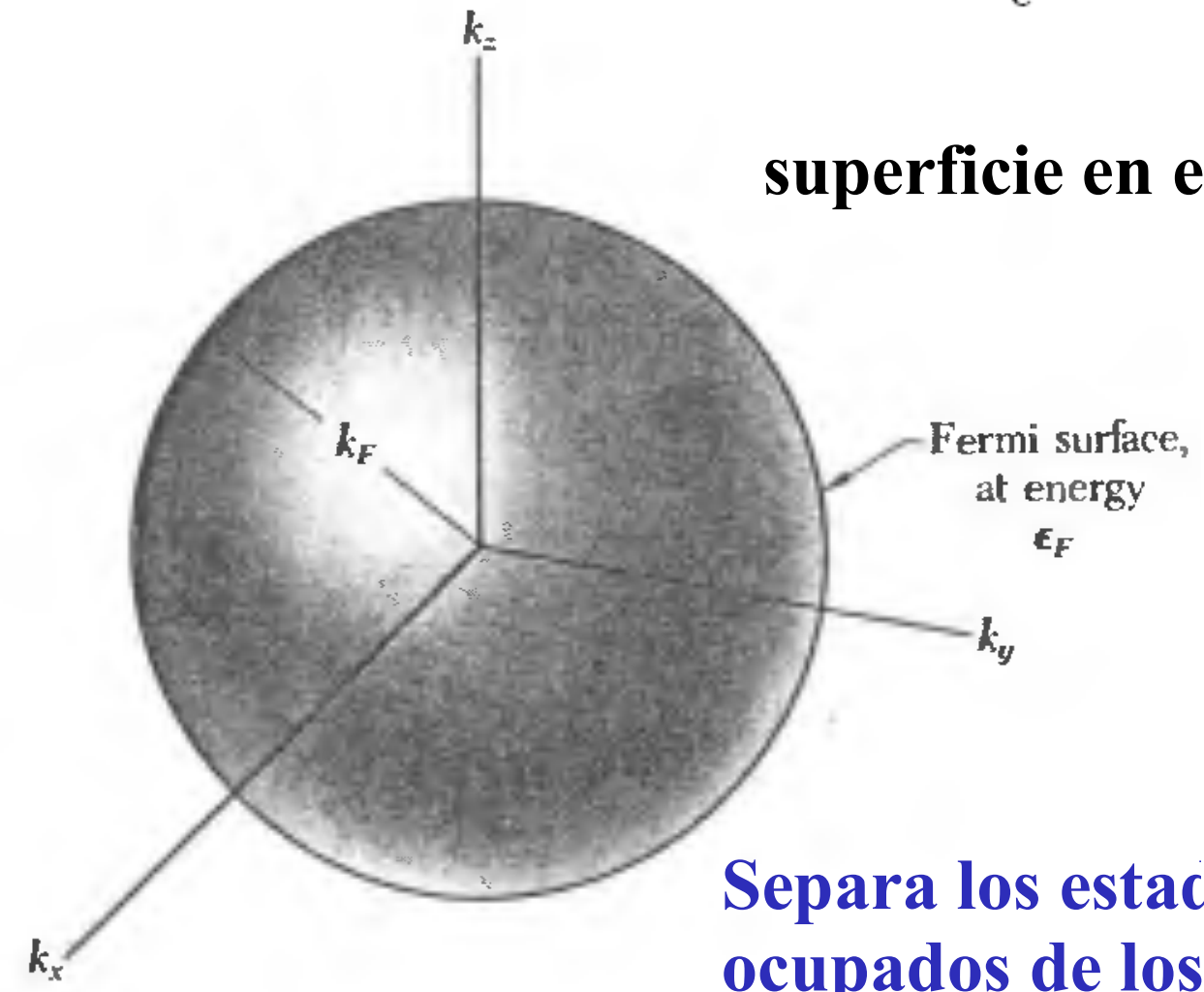
$$n_e = \frac{N_e}{V} = \frac{k_F^3}{3\pi^2}$$

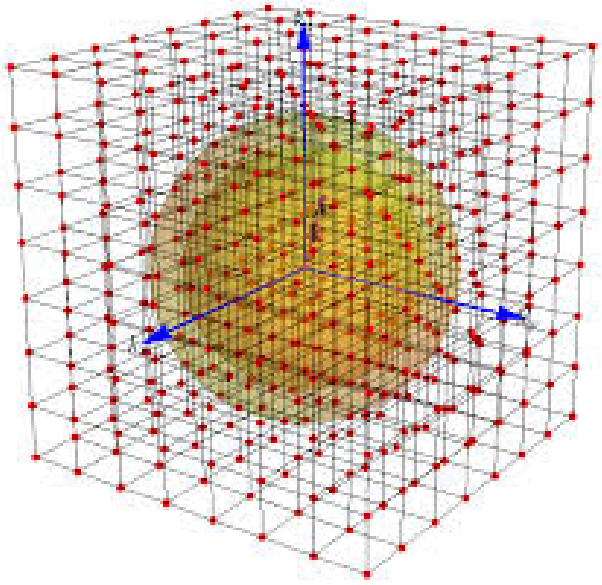
$$\varepsilon_F = \frac{\hbar^2}{2m_e}(3\pi^2 n_e)^{2/3}$$

Superficie de Fermi

$$\varepsilon_{\boldsymbol{k}} = \frac{\hbar^2 k^2}{2m_e} = \frac{\hbar^2}{2m_e}(k_x^2 + k_y^2 + k_z^2) \quad = \varepsilon_F$$

superficie en el espacio k (esfera)





Separa los estados ocupados de los vacíos

Metal	Electron concentration, in cm^{-3}	Radius[a] parameter r_s	Fermi wavevector, in cm^{-1}	Fermi velocity, in cm s^{-1}	Fermi energy, in eV	Fermi temperature $T_F \equiv \epsilon_F/k_B$, in deg K
Li	4.70×10^{22}	3.25	1.11×10^{8}	1.29×10^{8}	4.72	5.48×10^{4}
Na	2.65	3.93	0.92	1.07	3.23	3.75
K	1.40	4.86	0.75	0.86	2.12	2.46
Rb	1.15	5.20	0.70	0.81	1.85	2.15
Cs	0.91	5.63	0.64	0.75	1.58	1.83
Cu	8.45	2.67	1.36	1.57	7.00	8.12
Ag	5.85	3.02	1.20	1.39	5.48	6.36
Au	5.90	3.01	1.20	1.39	5.51	6.39
Be	24.2	1.88	1.93	2.23	14.14	16.41
Mg	8.60	2.65	1.37	1.58	7.13	8.27
Ca	4.60	3.27	1.11	1.28	4.68	5.43
Sr	3.56	3.56	1.02	1.18	3.95	4.58
Ba	3.20	3.69	0.98	1.13	3.65	4.24
Zn	13.10	2.31	1.57	1.82	9.39	10.90
Cd	9.28	2.59	1.40	1.62	7.46	8.66
Al	18.06	2.07	1.75	2.02	11.63	13.49
Ga	15.30	2.19	1.65	1.91	10.35	12.01
In	11.49	2.41	1.50	1.74	8.60	9.98
Pb	13.20	2.30	1.57	1.82	9.37	10.87
Sn(w)	14.48	2.23	1.62	1.88	10.03	11.64

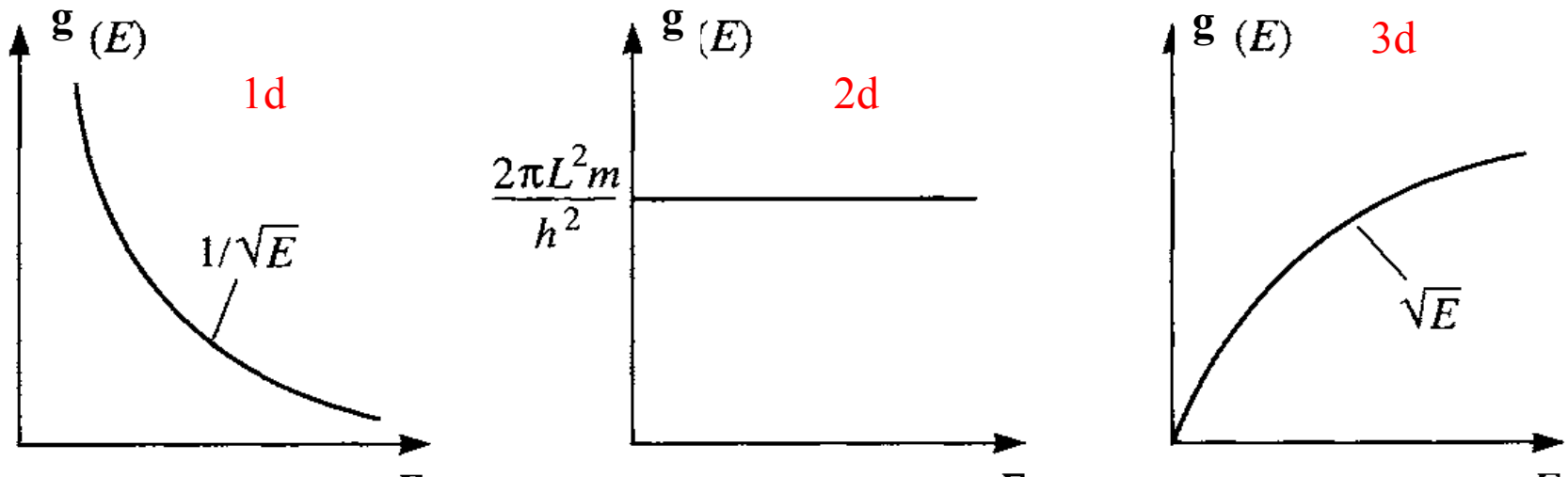
La energía total del estado fundamental del gas es:

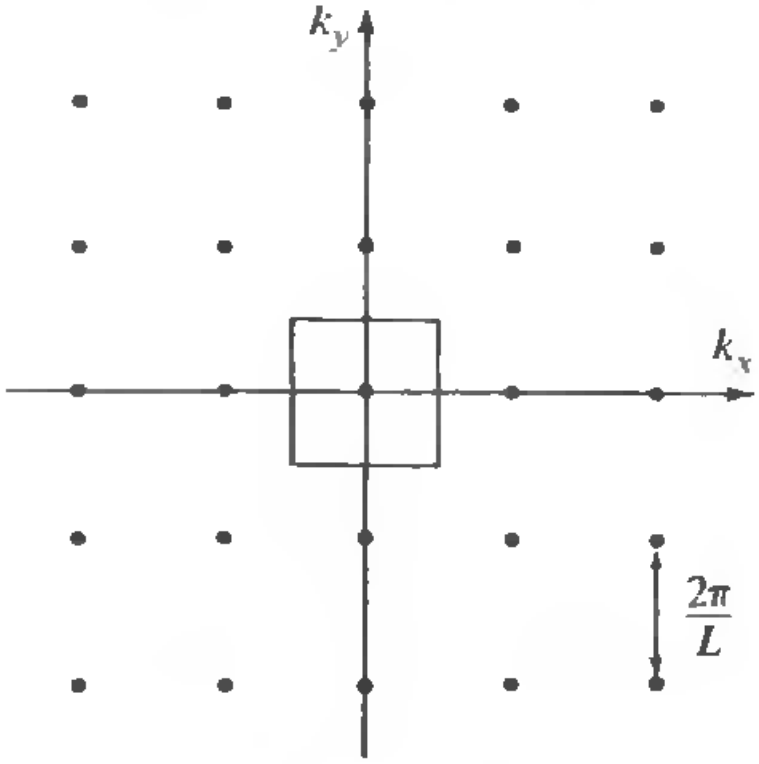
$$E_0 = \sum_{|\boldsymbol{k}|\leq k_F} \sum_{\sigma} \frac{\hbar^2 k^2}{2m_e} = 2 \sum_{|\boldsymbol{k}|\leq k_F} \frac{\hbar^2 k^2}{2m_e}$$

Para una muestra macroscópica, los valores permitidos de k llenan densamente el espacio

$$2\sum_{k} \rightarrow \int d^3k \rightarrow \int d\varepsilon \Longrightarrow$$

densidad de estados
g(k) g(E)





$$(2\pi/L)^3$$

Volumen del espacio k asociado a cada estado

El número de estados que contiene una región de volumen Ω es:

$$\frac{\Omega}{(2\pi/L)^3} = \frac{\Omega V}{8\pi^3} \Longrightarrow \frac{V}{8\pi^3}$$

Densidad de estados en el espacio k

$$E_0 = \sum_{|\boldsymbol{k}|\le k_\mathrm{F}} \sum_{\sigma} \frac{\hbar^2 k^2}{2m_\mathrm{e}} = 2 \sum_{|\boldsymbol{k}|\le k_\mathrm{F}} \frac{\hbar^2 k^2}{2m_\mathrm{e}} .$$

$$E_0 = 2\frac{V}{(2\pi)^3} \int_{|\boldsymbol{k}|\le k_\mathrm{F}} \frac{\hbar^2 k^2}{2m_\mathrm{e}} \,\mathrm{d}\boldsymbol{k} \longrightarrow \boxed{g(\boldsymbol{k}) = \frac{2}{(2\pi)^3}}$$

$$V \int g(\varepsilon)\, \varepsilon \,\mathrm{d}\varepsilon \longrightarrow g(\varepsilon) \quad ?$$

Para el gas de electrones en 3D:

$$g(\varepsilon)\,\mathrm{d}\varepsilon = \frac{2}{(2\pi)^3} 4\pi k^2 \,\mathrm{d}k \qquad \varepsilon = \frac{\hbar^2 k^2}{2m_\mathrm{e}}$$

Se obtiene: $$g(\varepsilon) = \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2}\sqrt{\varepsilon}$$

$$E_0 = \sum_{|\boldsymbol{k}|\le k_F}\sum_{\sigma}\frac{\hbar^2k^2}{2m_e} = 2\sum_{|\boldsymbol{k}|\le k_F}\frac{\hbar^2k^2}{2m_e}\,.$$

$$E_0 = V\int_0^{E_F} g(\varepsilon)\,\varepsilon\,\mathrm{d}\varepsilon$$

$$E_0 = 2\frac{V}{(2\pi)^3}\int_{|\boldsymbol{k}|\le k_F}\frac{\hbar^2k^2}{2m_e}\,\mathrm{d}\boldsymbol{k} = 2\frac{V}{(2\pi)^3}\int_0^{k_F}\frac{\hbar^2k^2}{2m_e}4\pi k^2\,\mathrm{d}k = \frac{V}{5\pi^2}k_F^3\frac{\hbar^2k_F^2}{2m_e}$$

$$\frac{N_e}{2} = \frac{4\pi k_F^3}{3}\frac{V}{(2\pi)^3} = \frac{k_F^3}{6\pi^2}V \quad\Longrightarrow\quad n_e = \frac{N_e}{V} = \frac{k_F^3}{3\pi^2}$$

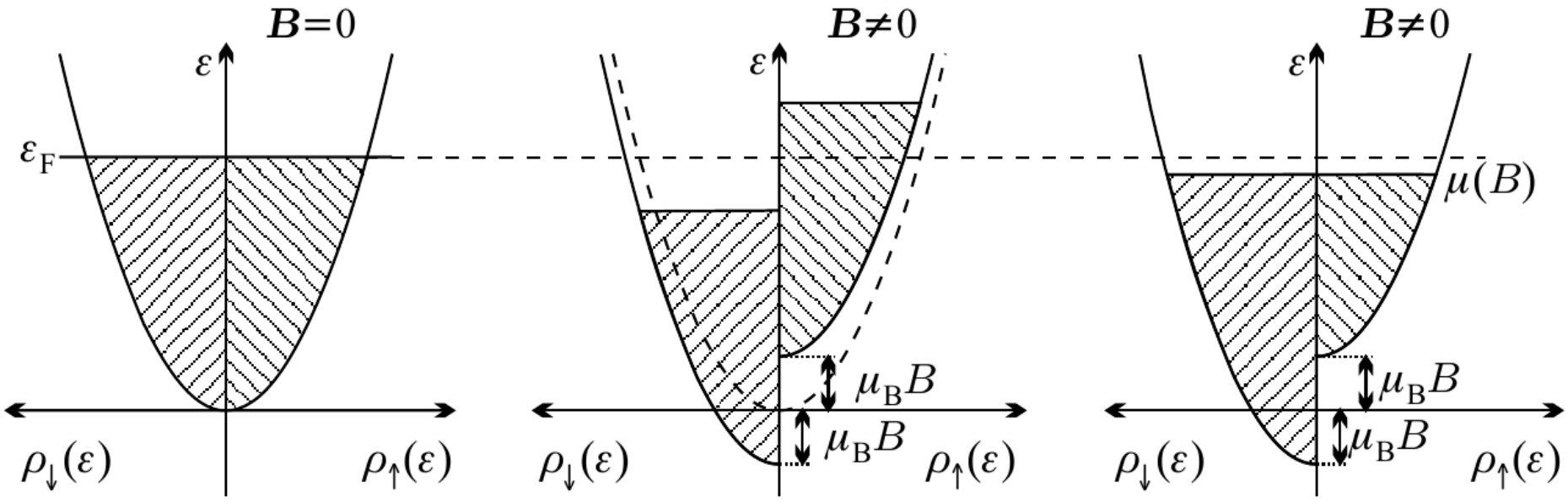
$$\varepsilon_F = \frac{\hbar^2k_F^2}{2m_e} = \frac{\hbar^2}{2m_e}(3\pi^2n_e)^{2/3} \qquad E_0 = \frac{3}{5}\frac{\hbar^2k_F^2}{2m_e}N_e = \frac{3}{5}\varepsilon_F N_e$$

Susceptibilidad de Pauli

Al aplicar un campo magnético al gas, los niveles de energía que estaban degenerados en spin se splitean

$$\varepsilon_{\boldsymbol{k}\sigma} = \varepsilon_{\boldsymbol{k}} - \tfrac{1}{2} g_{\mathrm{e}} \mu_{\mathrm{B}} B \sigma \qquad \sigma = \pm 1$$

$$\uparrow \ \varepsilon_{\boldsymbol{k}\sigma} = \varepsilon_{\boldsymbol{k}} - \tfrac{1}{2} g_{\mathrm{e}} \mu_{\mathrm{B}} B \qquad \downarrow \ \varepsilon_{\boldsymbol{k}\sigma} = \varepsilon_{\boldsymbol{k}} + \tfrac{1}{2} g_{\mathrm{e}} \mu_{\mathrm{B}} B$$

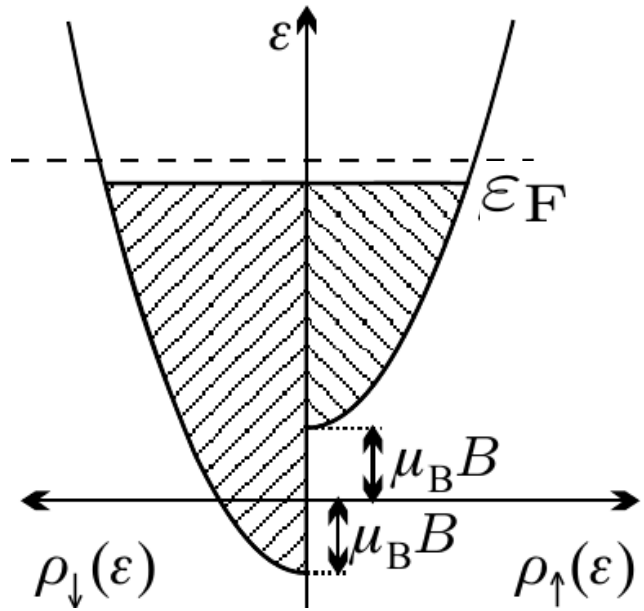


$$M = \tfrac{1}{2} g_e \mu_B \left(n_\uparrow - n_\downarrow\right)$$

$$n_\uparrow + n_\downarrow = n_e$$

$$n_{\uparrow\downarrow} = \int \rho_{\uparrow\downarrow}(\varepsilon)\ \mathrm{d}\varepsilon$$

Extremos de integración



$$M = \tfrac{1}{4} g_e^2 \mu_B^2 B \rho(\varepsilon_F)$$

Recordando que $B = \mu_0 H$

$$\chi_P = \tfrac{1}{4} \mu_0 (g_e \mu_B)^2 \rho(\varepsilon_F)$$

Pauli susceptibility

$$\chi_P = \frac{3}{8} n_e \frac{\mu_0 (g_e \mu_B)^2}{\varepsilon_F} = \frac{3}{2} n_e \frac{\mu_0 (g_e \mu_B)^2}{4 k_B T_F}$$

Calor específico de metales

La energía interna de un gas clásico de N electrones es:

$U(T) = 3/2\ N\ K\ T$ (1/2 K T por grado de libertad)

C_v (clas) = $\partial E/\partial T = 3/2\ N\ K$

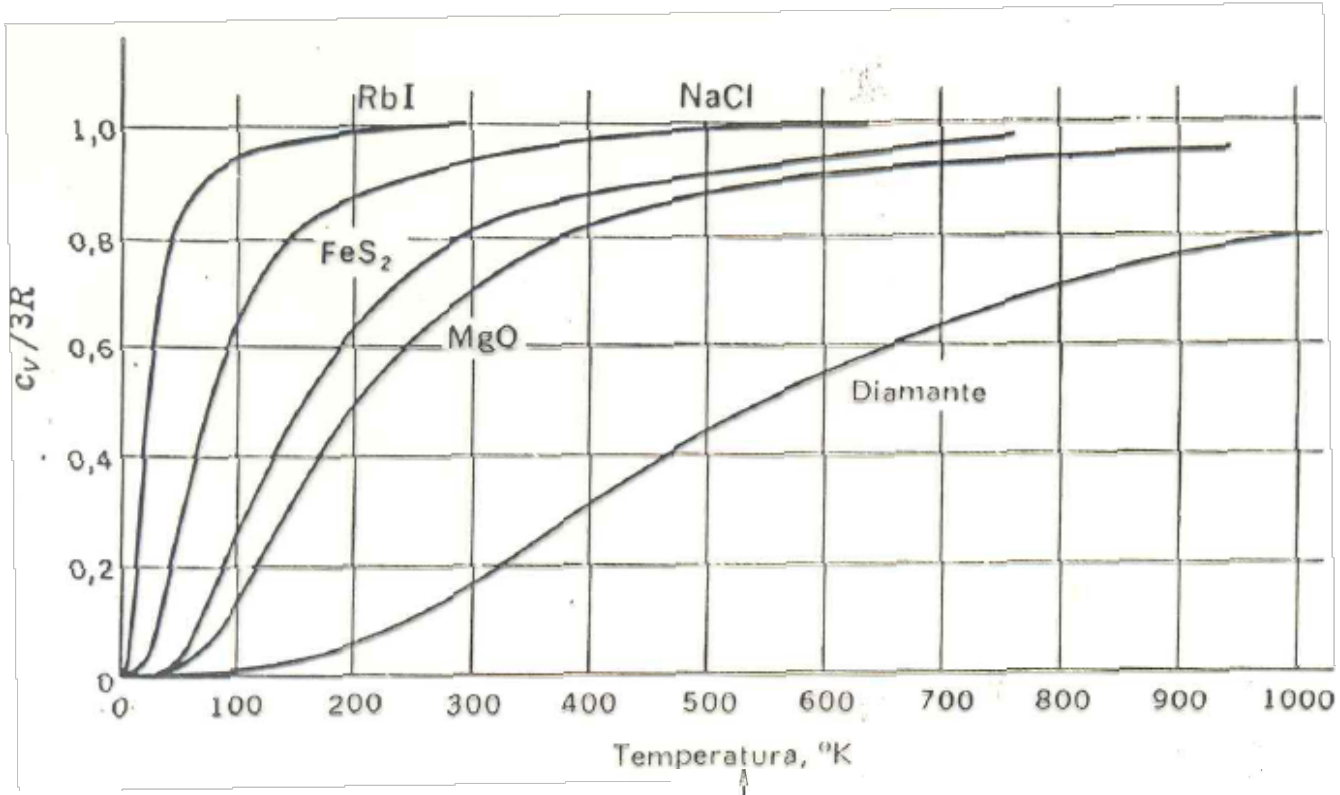
Si $N = N_A$ ($6.023\ 10^{23}$), el calor específico molar es:

C_v (clas) = $3/2\ N_A\ K = 3/2\ R$

Este término debería sumarse a la contribución proveniente de las vibraciones de la red. El C_v de un metal monoatómico a temperatura ambiente sería

$$C_v = 3/2\ R + 3\ R = 9/2\ R$$

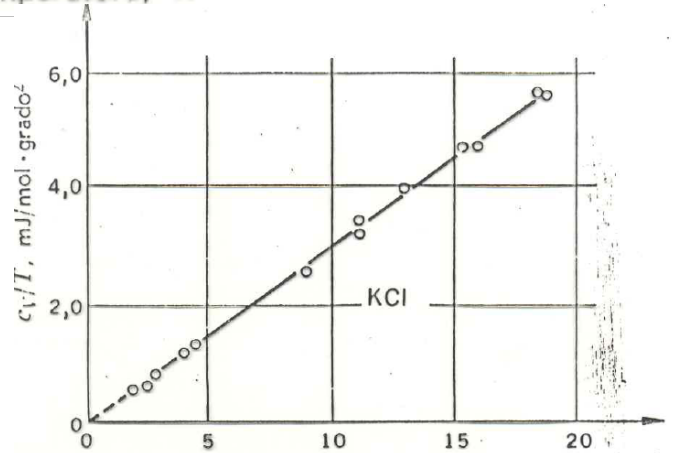
Contribución de los fonones (aisladores)



A bajas temperaturas

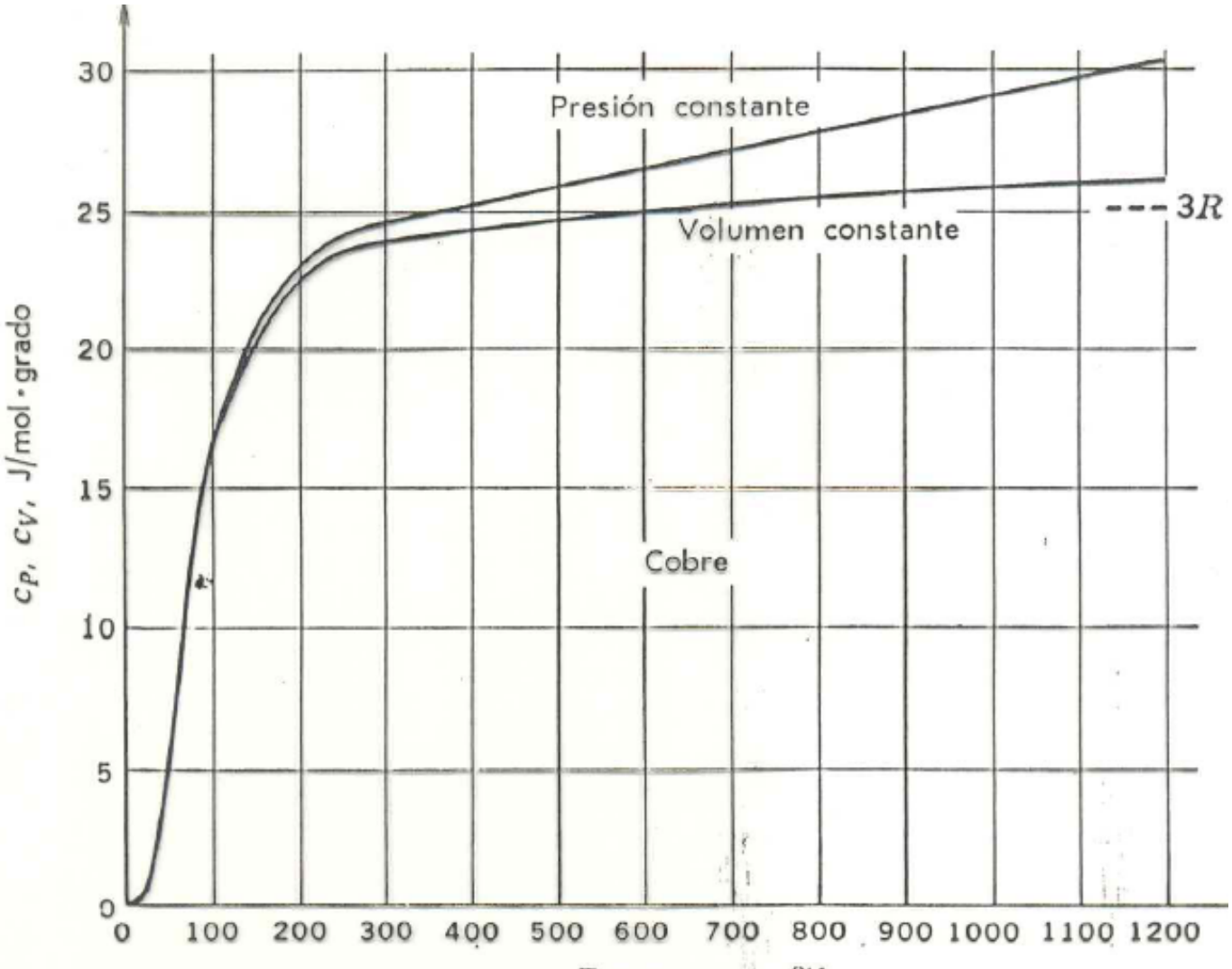
$$C_v = \alpha\, T^3$$

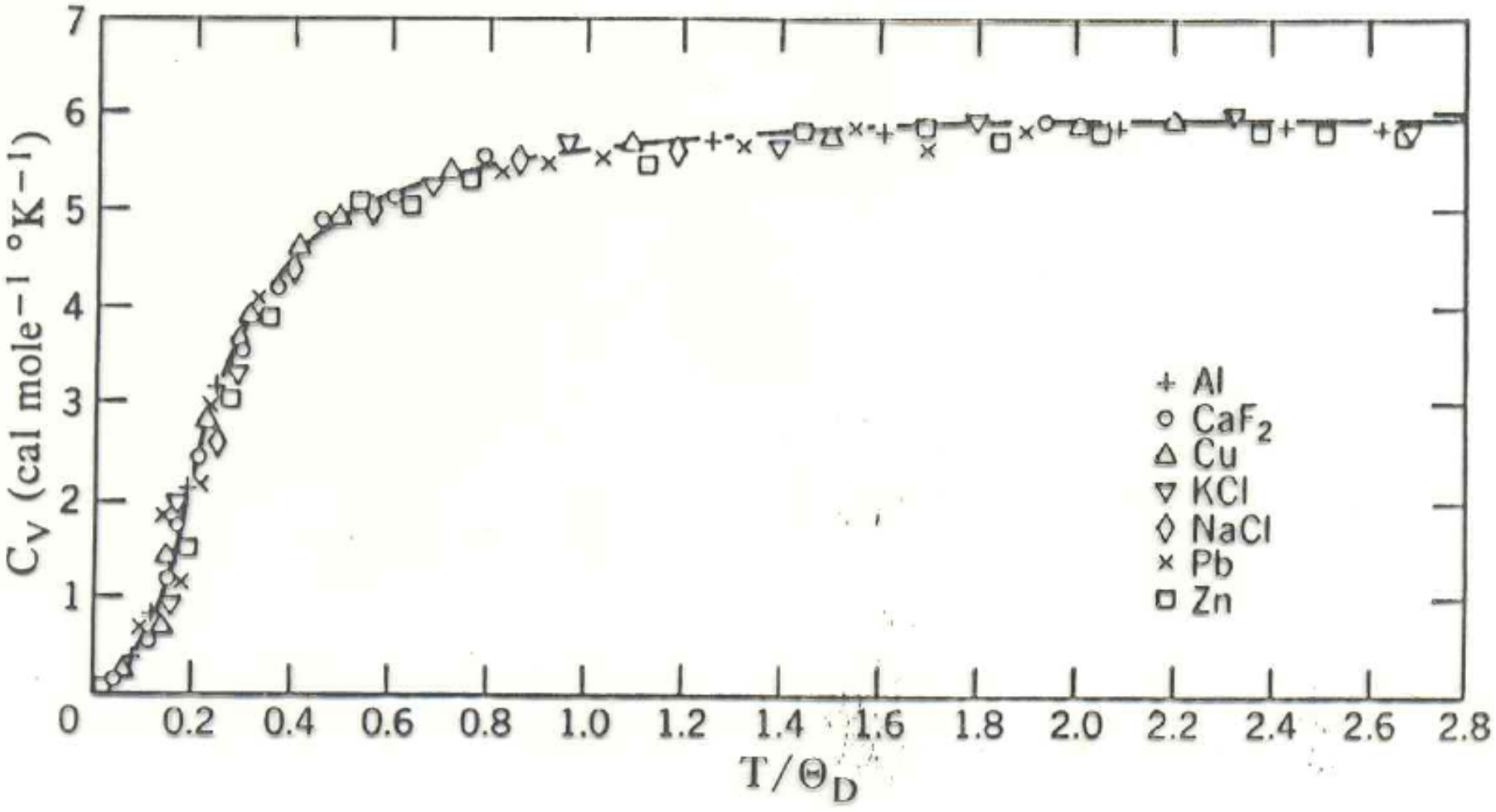
$$C_v / T = \alpha\, T^2$$



En un metal ? $C_v = C_v$ (fonones) + 3/2 R ?

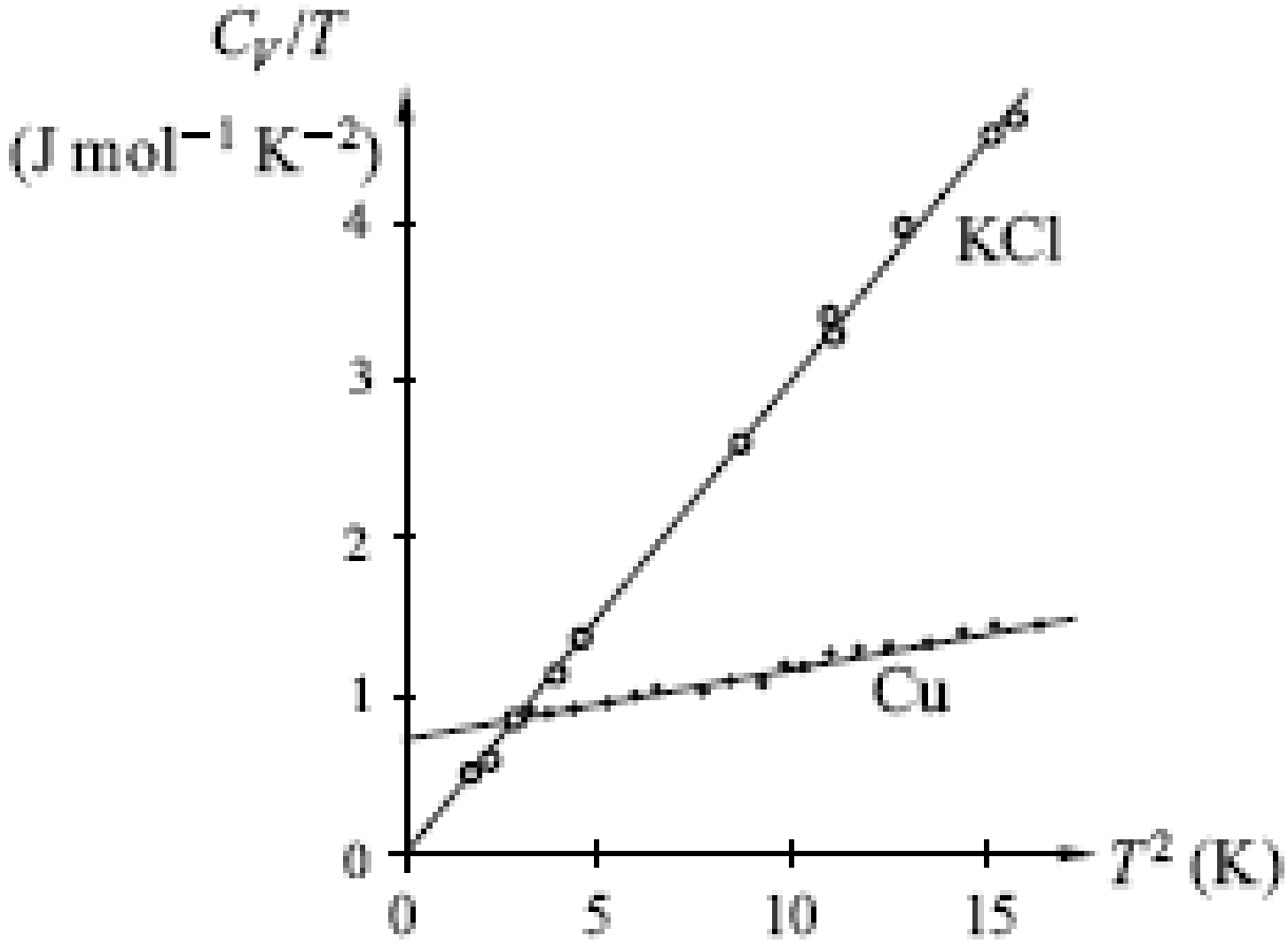
$T=T_{amb} \longrightarrow C_v = 3\,R + 3/2\,R = 9/2\,R$?





La contribución de los electrones de conducción al calor especifico de un metal es pequeña a temperatura ambiente, comparada con la contribución proveniente de las vibraciones de la red (fonones)

A muy bajas temperaturas :



Tratamiento cuántico - Gas de Sommerfeld

La energía total del estado fundamental del gas es :

$$E_0 = \sum_{|\boldsymbol{k}| \le k_\mathrm{F}} \sum_{\sigma} \frac{\hbar^2 k^2}{2m_\mathrm{e}} = 2 \sum_{|\boldsymbol{k}| \le k_\mathrm{F}} \frac{\hbar^2 k^2}{2m_\mathrm{e}}$$

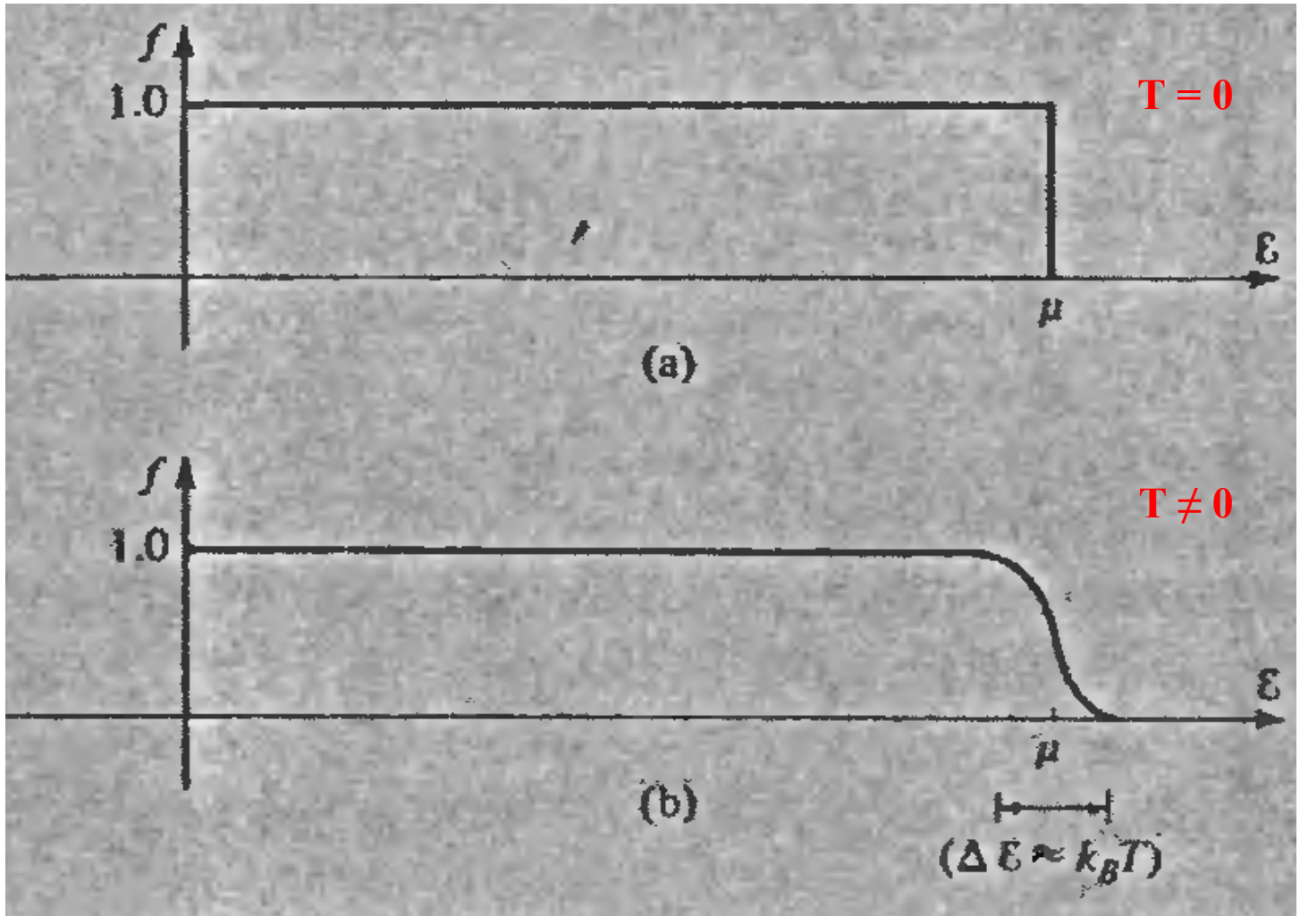
E(T) $\Longrightarrow$ **$C_v = \partial E/\partial T$**

Cómo calculamos la energía interna E(T) ?

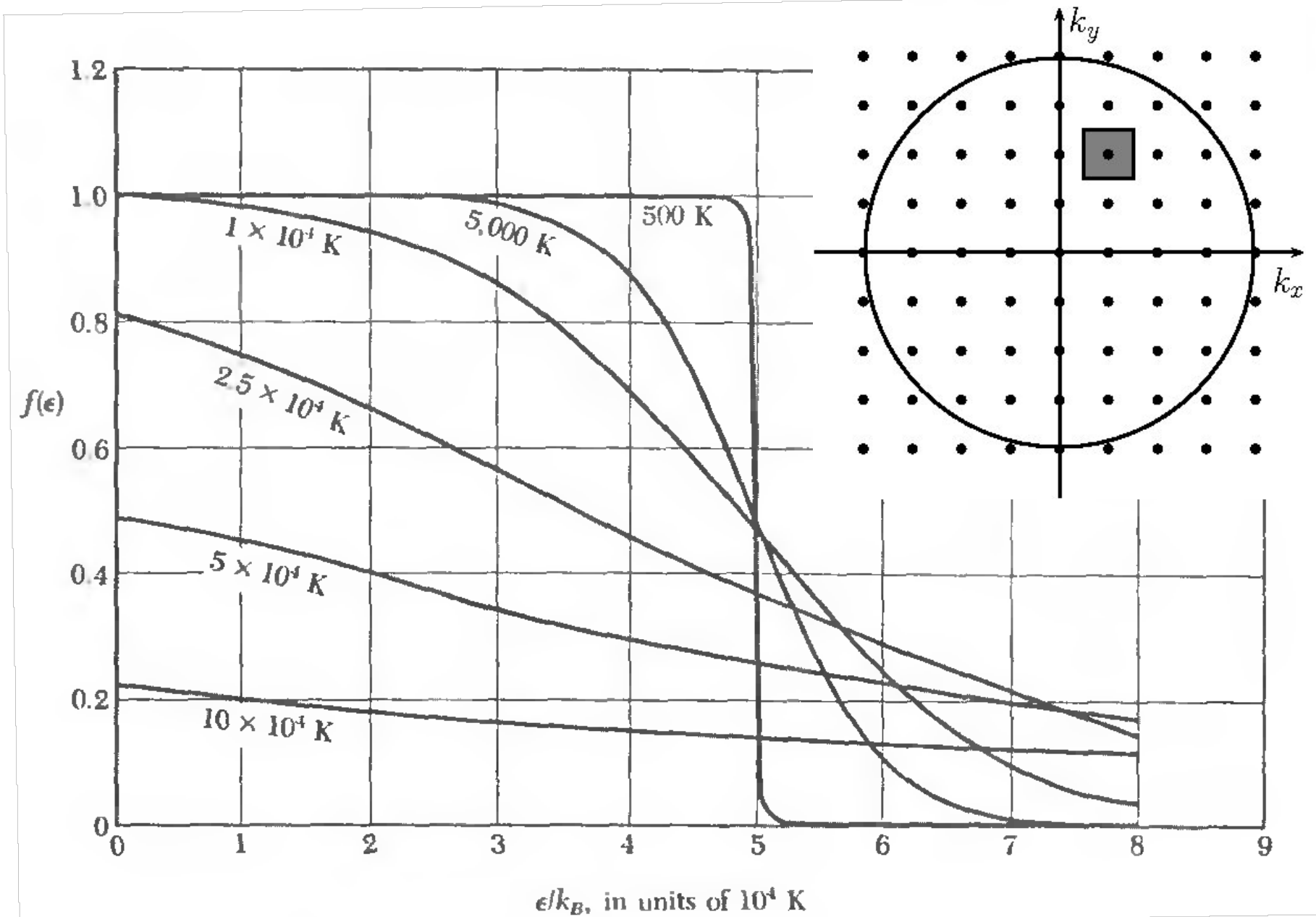
$$E = 2\sum_{\boldsymbol{k}} \varepsilon_{\boldsymbol{k}} f_0(\varepsilon_{\boldsymbol{k}}) = 2\frac{V}{(2\pi)^3} \int \varepsilon_{\boldsymbol{k}} f_0(\varepsilon_{\boldsymbol{k}}) \mathrm{d}\boldsymbol{k} = V \int \varepsilon f_0(\varepsilon) \rho(\varepsilon)\, \mathrm{d}\varepsilon$$

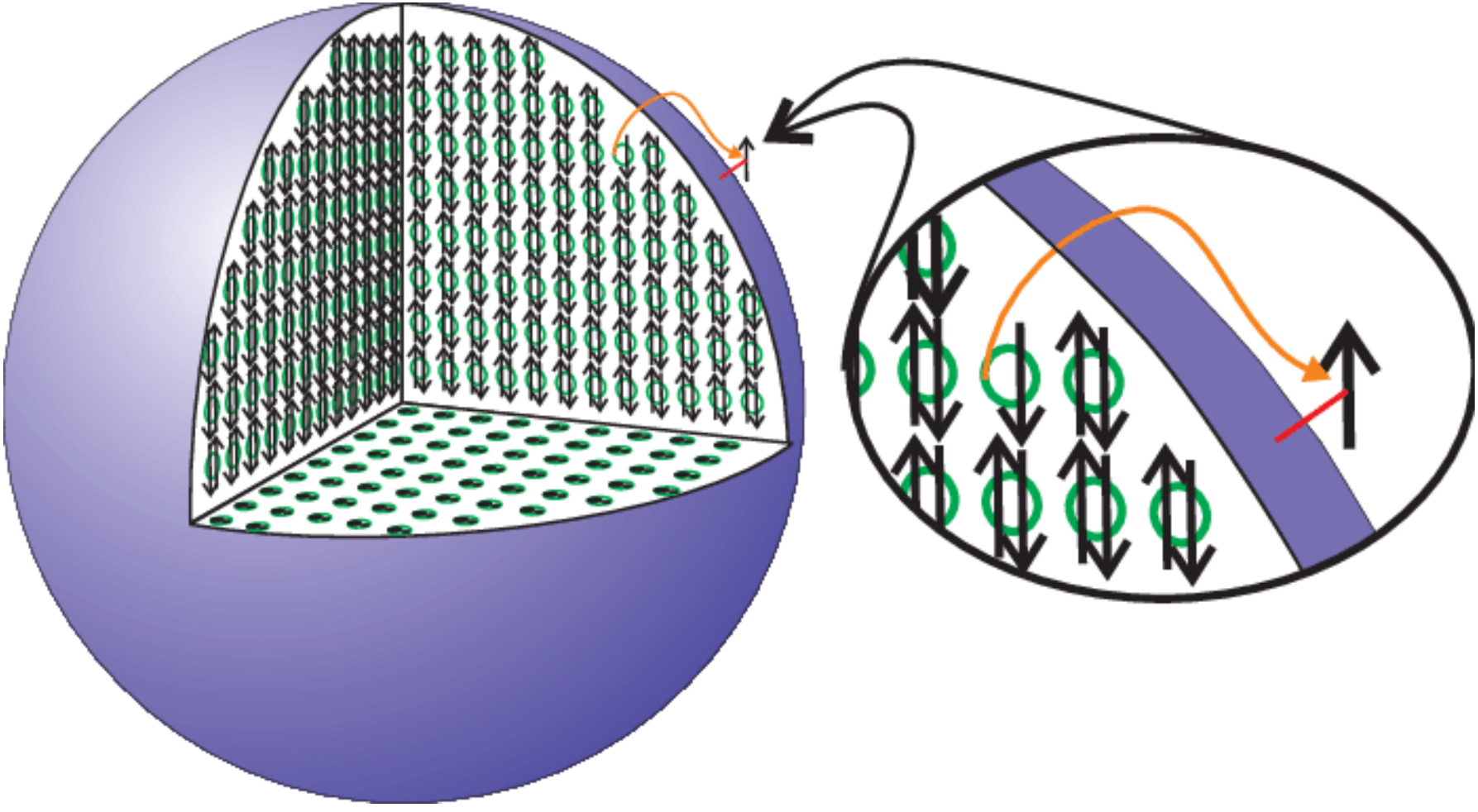
donde $$f_0(\varepsilon) = \frac{1}{\exp[(\varepsilon - \mu)/k_\mathrm{B}T] + 1}$$

Distribución de Fermi-Dirac

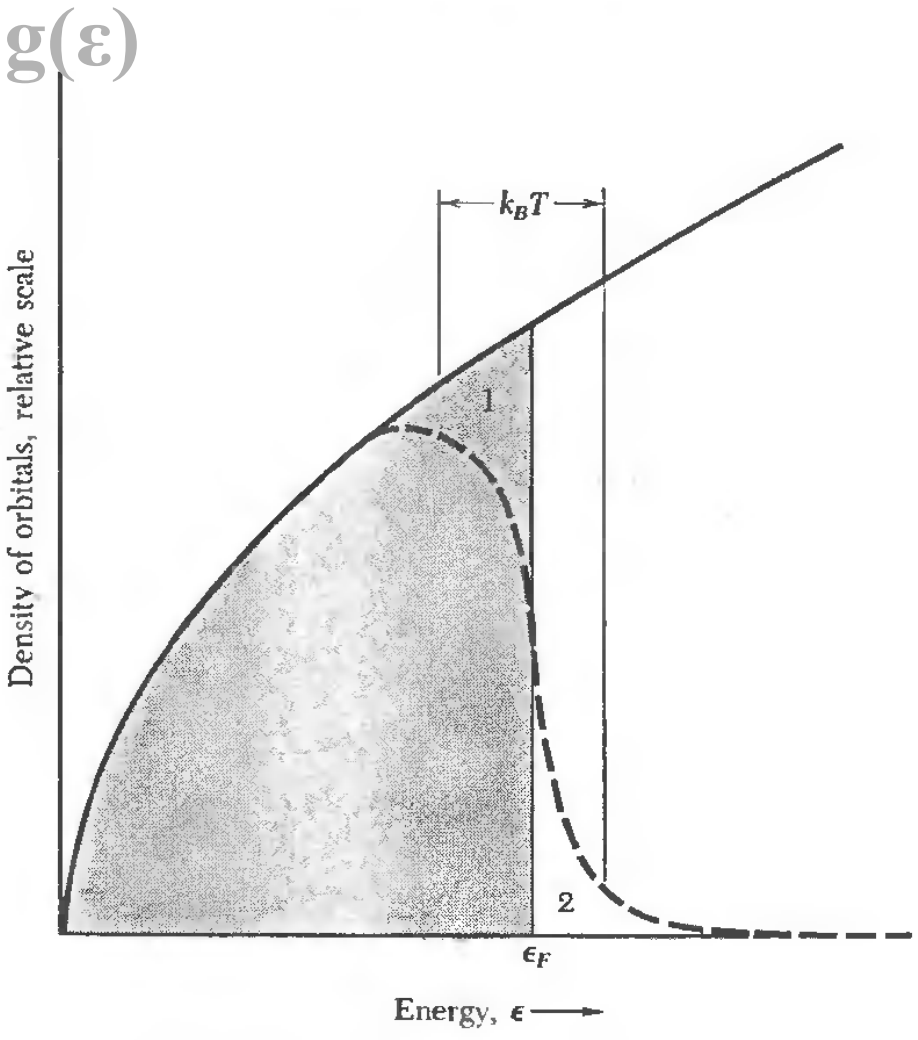


$$E_F = \lim_{T \to 0} \mu$$


1.2
1.0
0.8
0.6
0.4
0.2
0
$f(\epsilon)$
1×10^4 K
5,000 K
500 K
2.5×10^4 K
5×10^4 K
10×10^4 K
0
1
2
3
4
5
6
7
8
9
ϵ/k_B, in units of 10^4 K
k_y
k_x



Estimación de la contribución electrónica al C_v



$$\frac{\text{Nº } e^- \text{ excitados}}{V} \approx g(E_F)\, K\, T$$

Energía de excitación ≈ K T

$$\frac{E}{V} \approx g(E_F)\,(K\,T)^2$$

$$C_v \approx g(E_F)\, K^2\, T$$

$$c_{\mathrm{el}} = \frac{1}{V}\frac{\partial E}{\partial T} = \frac{\pi^2}{3}\rho(\varepsilon_{\mathrm{F}})k_{\mathrm{B}}^2 T$$

Se muestra que para un gas de electrones en 3D :

$$g(E_F) = \frac{3\,n}{2\,E_F} = \frac{3\,n}{2\,K\,T_F}$$

$$C_v \approx g(E_F)\,K^2\,T = 3/2\;n\,K\,T/T_F$$

$$C_v(clas) = 3/2\;n\,K$$

$$\frac{C_v}{C_v(clas)} \approx T/T_F \approx 10^{-2}$$

T=Tamb

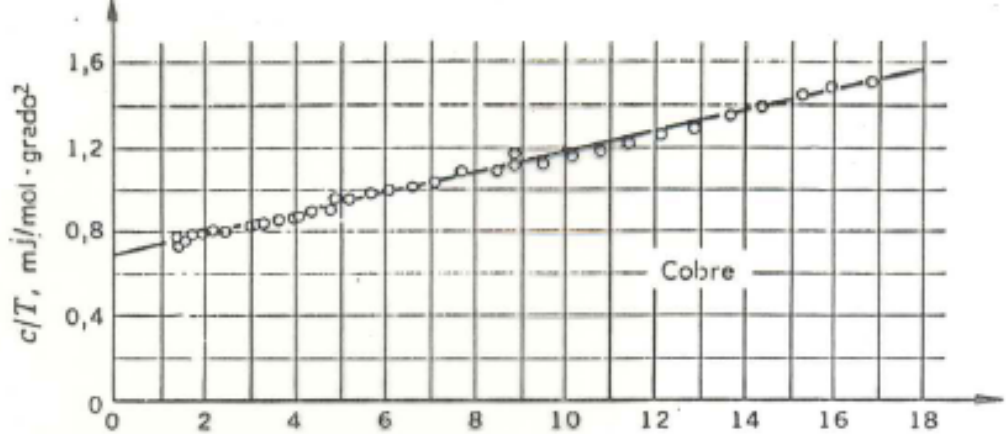
La contribución de los electrones de conducción al calor especifico de un metal a T_{amb} es ~1% del valor clásico

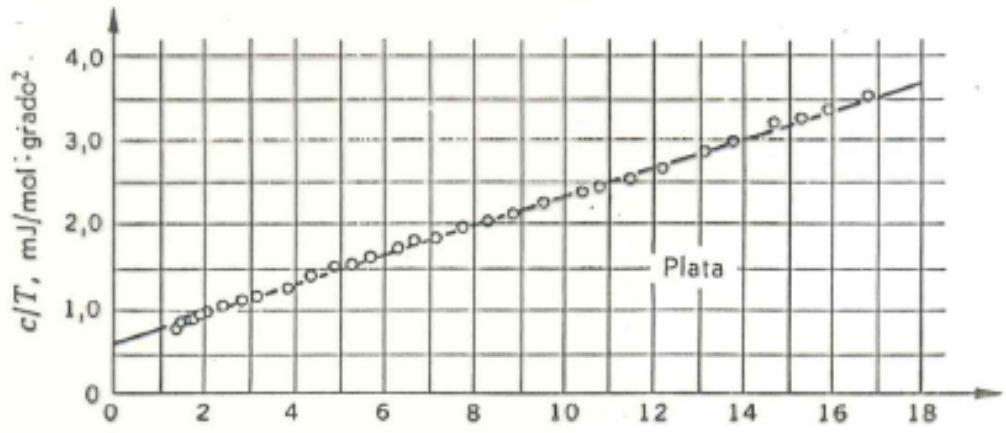
Para un metal a bajas temperaturas

$$C_v = C_v(e) + C_v(ph) = \gamma T + \alpha T^3$$

$$C_v / T = \gamma + \alpha T^2$$

γ es proporcional a $g(E_F)$





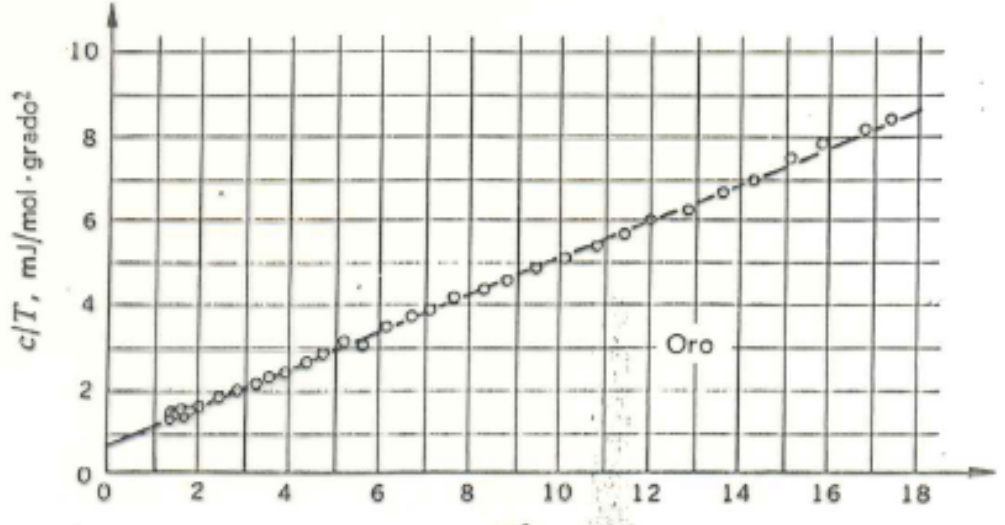


Table 2 Experimental and free electron values of electronic heat capacity constant γ of metals

(From compilations kindly furnished by N. Phillips and N. Pearlman. The thermal effective mass is defined by Eq. (38).

Observed γ in mJ mol^{-1} K^{-2}.
Calculated free electron γ in mJ mol^{-1} K^{-2}
m_{th}/m = (observed γ)/(free electron γ).

Li	Be											B	C	N
1.63	0.17													
0.749	0.500													
2.18	0.34													
Na	Mg											Al	Si	P
1.38	1.3											1.35		
1.094	0.992											0.912		
1.26	1.3											1.48		
K	Ca	Sc	Ti	V	Cr	Mn(γ)	Fe	Co	Ni	Cu	Zn	Ga	Ge	As
2.08	2.9	10.7	3.35	9.26	1.40	9.20	4.98	4.73	7.02	0.695	0.64	0.596		0.19
1.668	1.511									0.505	0.753	1.025		
1.25	1.9									1.38	0.85	0.58		
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn (w)	Sb
2.41	3.6	10.2	2.80	7.79	2.0	—	3.3	4.9	9.42	0.646	0.688	1.69	1.78	0.11
1.911	1.790									0.645	0.948	1.233	1.410	
1.26	2.0									1.00	0.73	1.37	1.26	
Cs	Ba	La	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg(α)	Tl	Pb	Bi
3.20	2.7	10.	2.16	5.9	1.3	2.3	2.4	3.1	6.8	0.729	1.79	1.47	2.98	0.008
2.238	1.937									0.642	0.952	1.29	1.509	
1.43	1.4									1.14	1.88	1.14	1.97	

Limite clásico ????