

# Introducción a la Física Nuclear 2024

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## Taller numérico sobre Campo Medio

Objetivo:

Utilizar un código que resuelve la ecuación de Schroedinger para una partícula para calcular el espectro de energía (funciones de onda y root-mean-square-radius) de un núcleo con un nucleón (protón o neutrón) de valencia.

# Revisión de modelo de partícula independiente

# Modelo de partícula independiente

Hamiltoniano de muchos cuerpos

$$H = \sum_{i=1}^A \left[ -\frac{\hbar^2}{2m_i} \right] \nabla_{\mathbf{r}_i}^2 + \sum_{i<j=1}^A v(\mathbf{r}_i, \mathbf{r}_j) \longrightarrow H = H_0 + V_{res}$$

Partículas independientes

$$H_0 = \sum_i^A h(\mathbf{r}_i)$$

$$h(\mathbf{r})\phi_\alpha(\mathbf{r}) = \varepsilon_\alpha\phi_\alpha(\mathbf{r})$$

$$H_0\Psi(\mathbf{r}_1, \dots, \mathbf{r}_A) = E\Psi(\mathbf{r}_1, \dots, \mathbf{r}_A)$$

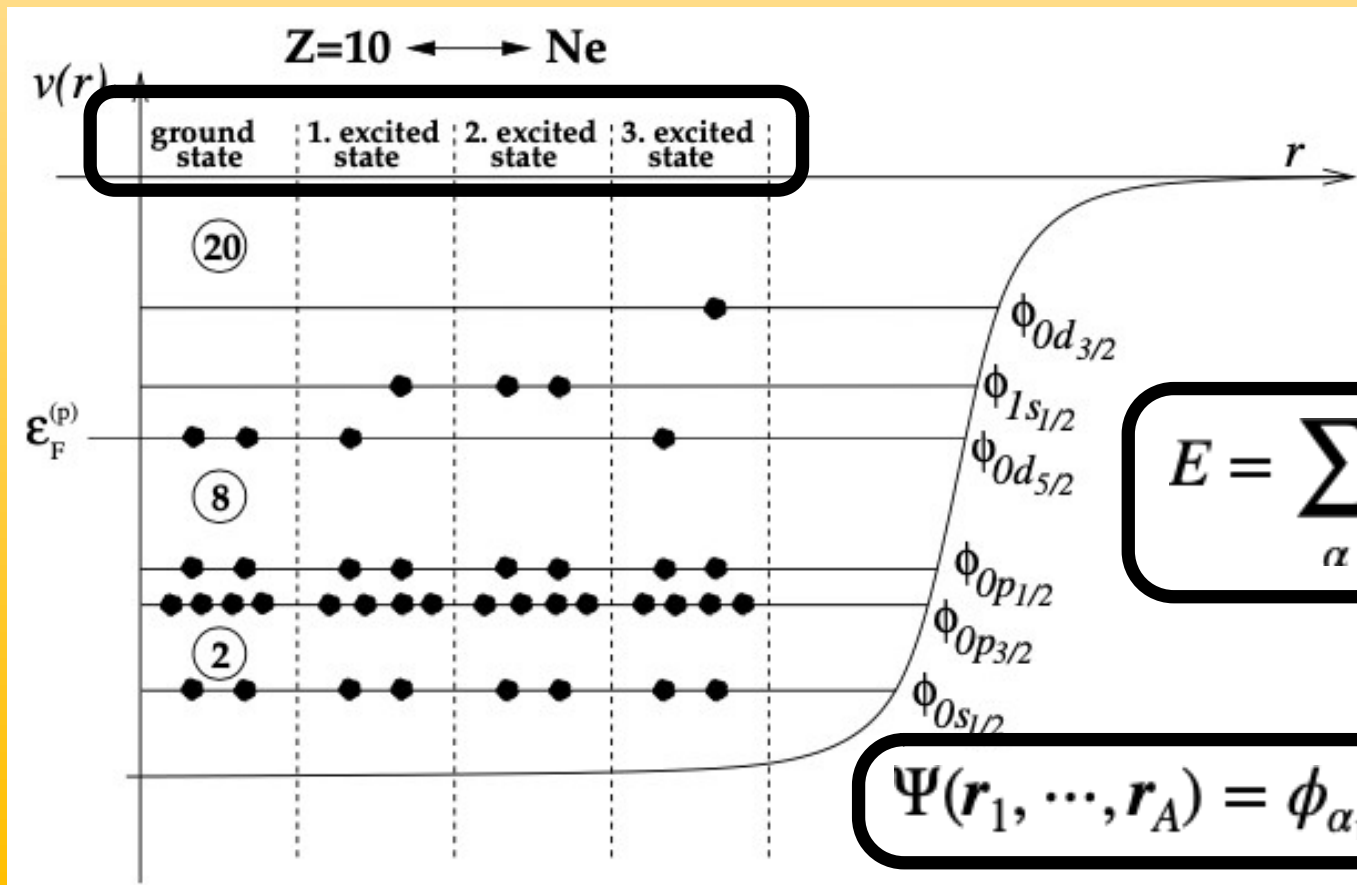
$$\Psi(\mathbf{r}_1, \dots, \mathbf{r}_A) = \phi_{\alpha_1}(\mathbf{r}_1)\cdots\phi_{\alpha_A}(\mathbf{r}_A)$$

# Modelo de partícula independiente: estados fundamental y excitados

Partículas independientes

$$H_0 = \sum_i^A h(\mathbf{r}_i)$$

$$h(\mathbf{r})\phi_\alpha(\mathbf{r}) = \varepsilon_\alpha\phi_\alpha(\mathbf{r})$$



$$\Psi(\mathbf{r}_1, \dots, \mathbf{r}_A) = \phi_{\alpha_1}(\mathbf{r}_1) \cdots \phi_{\alpha_A}(\mathbf{r}_A)$$

# Revisión de la ecuación diferencial radial

# Solución numérica

$$h(\mathbf{r}) \phi_{nljm}(\mathbf{r}) = \varepsilon_{nlj} \phi_{nljm}(\mathbf{r})$$

$$\phi_{nljm}(r, \theta, \phi) = \frac{u_{nlj}(r)}{r} \mathcal{Y}_{ljm}(\theta, \phi)$$

$$h(\mathbf{r}) = -\frac{\hbar^2}{2m} \nabla_r^2 + V(r) + V_{so}(r) \mathbf{l} \cdot \mathbf{s}$$

$$V(r) = V_{ws}(r) + V_{coul}(r)$$

$$h_{lj}(r) u_{nlj}(r) = \varepsilon_{nlj} u_{nlj}(r)$$

$$|l - 1/2| \leq j \leq l + 1/2$$

$$j = l \mp 1/2$$

$$h_{lj}(r) = -\frac{\hbar^2}{2\mu} \left[ \frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} \right] + V(r) + V_{so}(r) \frac{\hbar^2}{2} \left[ j(j+1) - l(l+1) - \frac{3}{4} \right]$$

$$\frac{1}{\mu} = \frac{1}{m_{\text{proyectil}}} + \frac{1}{m_{\text{blanco}}}$$

$$j = l + 1/2$$

$$j(j+1) - l(l+1) - \frac{3}{4} = (l + 1/2)(l + 1/2 + 1) - l(l+1) - 3/4$$

$$= (l + 1/2)(l + 3/2) - l(l+1) - 3/4$$

$$= 2l + 3/4 - l - 3/4 = l$$

# Solución numérica: Ecuaciones

$$R = e^{-\lambda r}$$

$$\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} u_{nlj}(r) + \left[ \epsilon_{nlj} - V_{ws}(r) - \cancel{\xi_{lj} V_{so}(r)} - \frac{\hbar^2}{2\mu} \frac{l(l+1)}{r^2} \right] u_{nlj}(r) = 0$$

$$\xi_{l,j=l+1/2} = l$$

$$\xi_{l,j=l-1/2} = -(l+1)$$

$$V_{ws}(r) = \frac{-V_{ws}}{1 + e^{\frac{r-R}{a}}} \quad V_{ws} > 0$$

$$V_{so}(r) = \frac{-V_{so}}{ra} \frac{2}{\hbar^2} \frac{e^{\frac{r-R}{a}}}{(1 + e^{\frac{r-R}{a}})^2} \quad V_{so} > 0$$

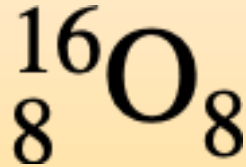
$$u_{nlj}(r) \rightarrow 0 \quad r \rightarrow \infty$$



**Revisión sobre  
espectro experimental  
versus  
Modelo de Capas para  
un nucleón**

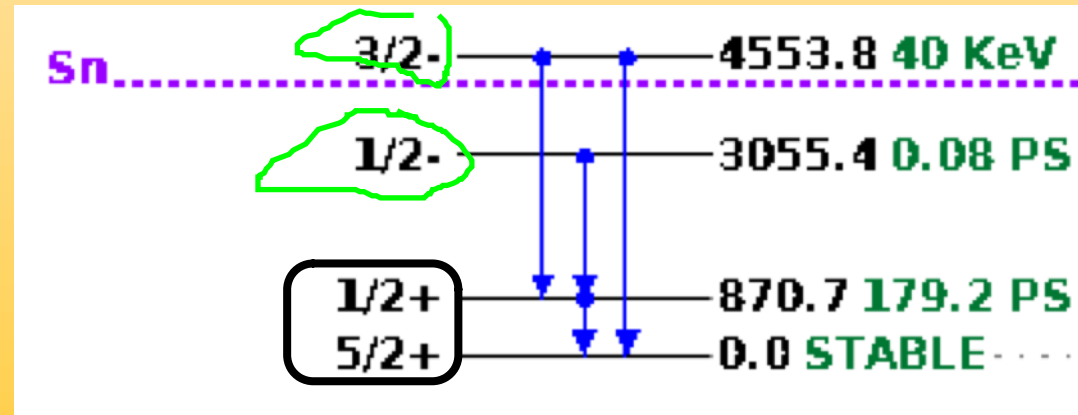
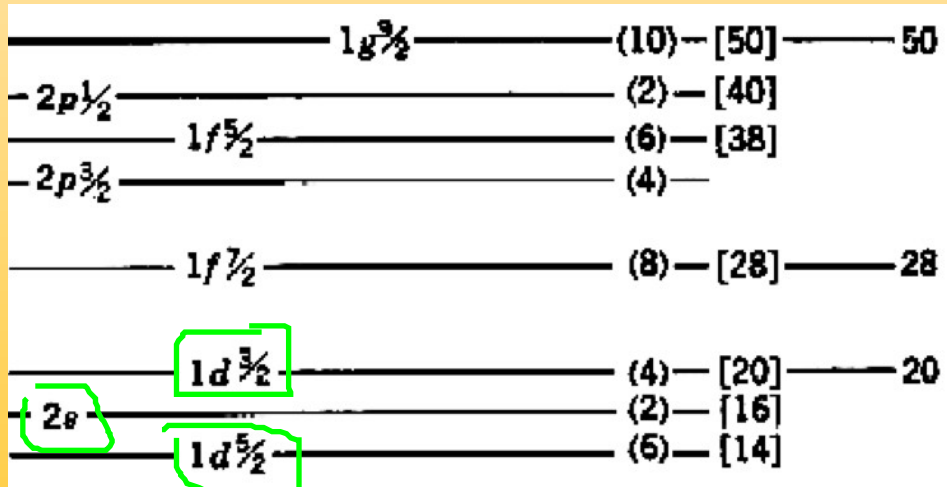


# Espectro $^{17}\text{O}$



Un neutrón en un núcleo mágico

## VALENCIA



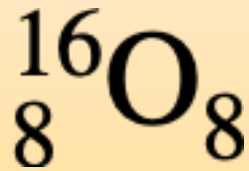
## CAROZO



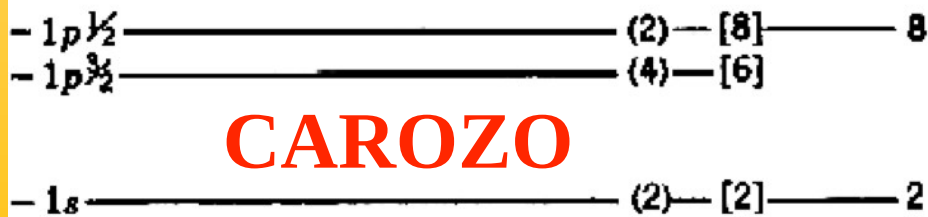
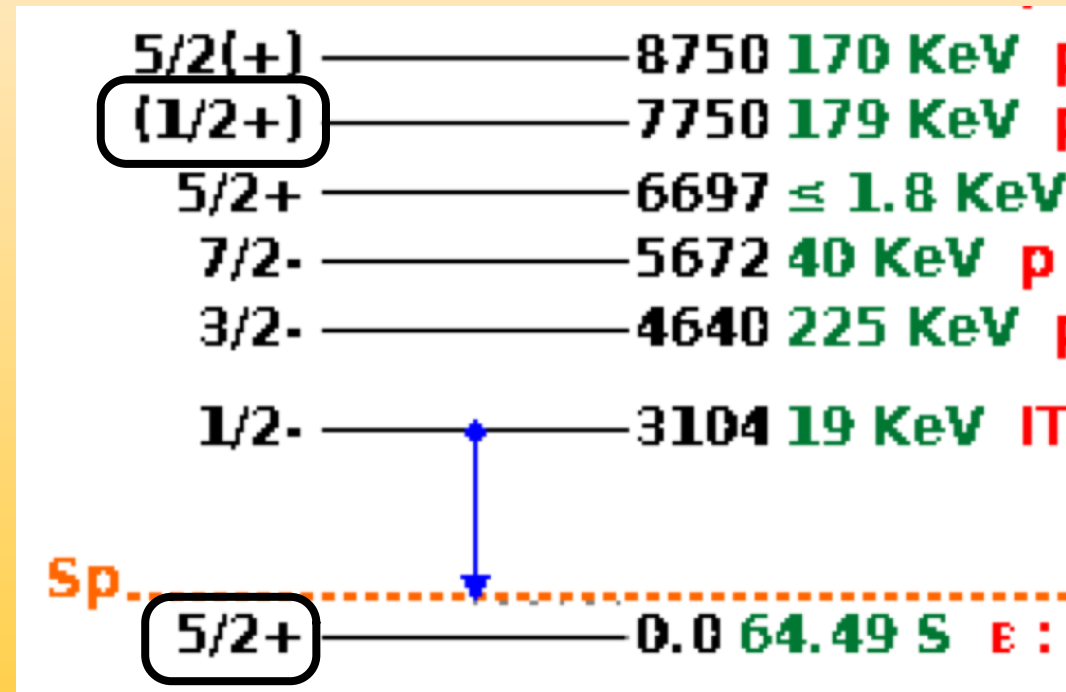
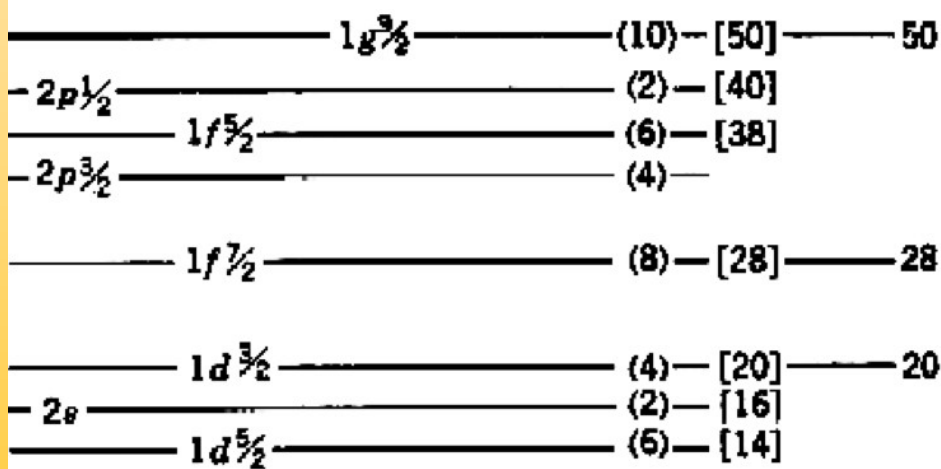
Núcleo doblemente mágico

# Espectro $^{17}\text{F}$

Un protón en un núcleo mágico



## VALENCIA

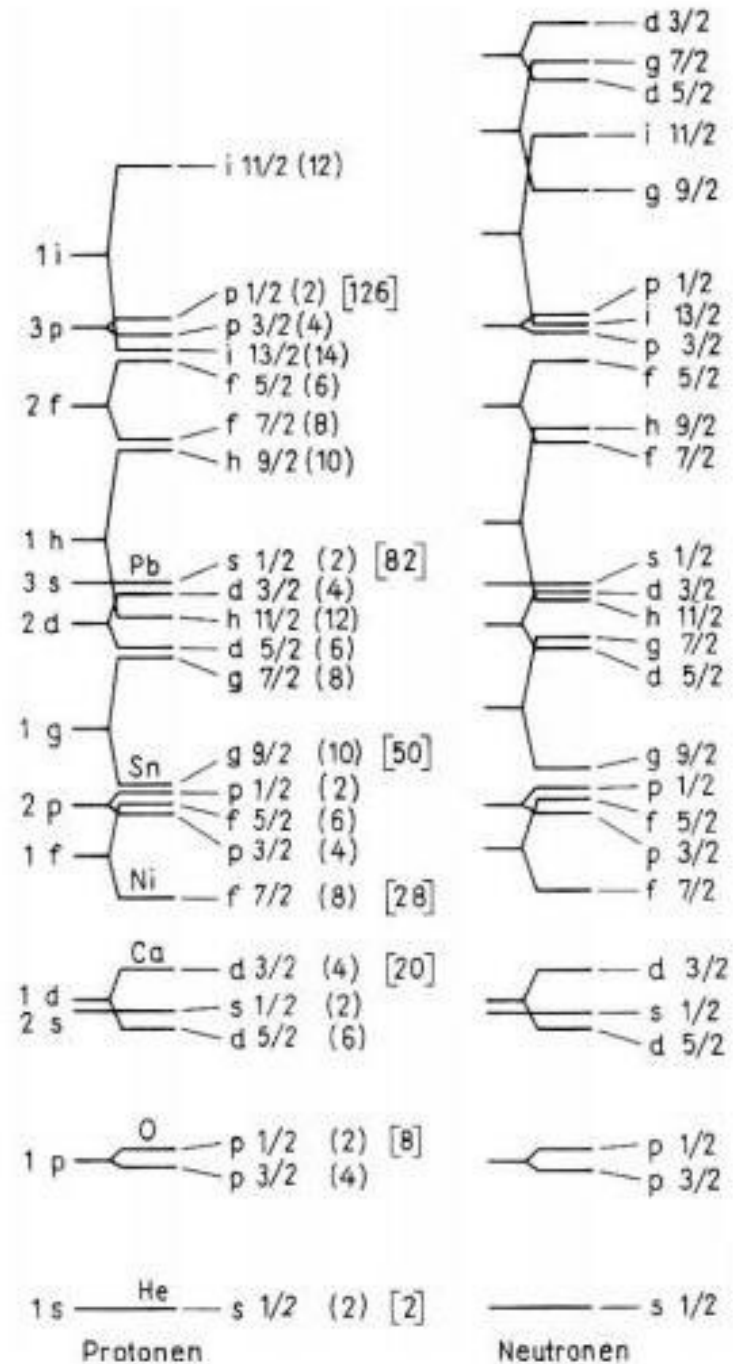


## CAROZO



Núcleo doblemente mágico

# Diferencia entre los niveles de protones y neutrones

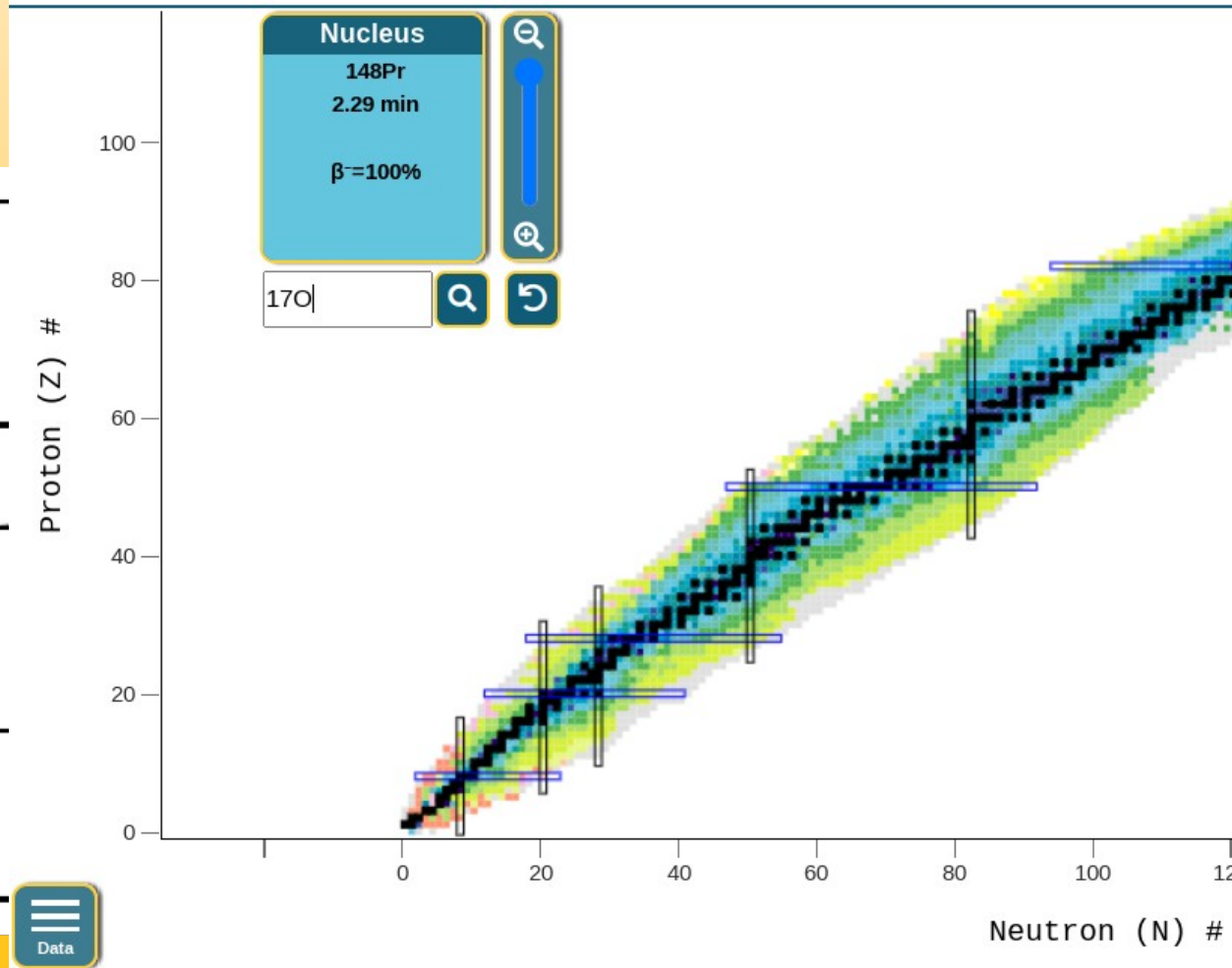
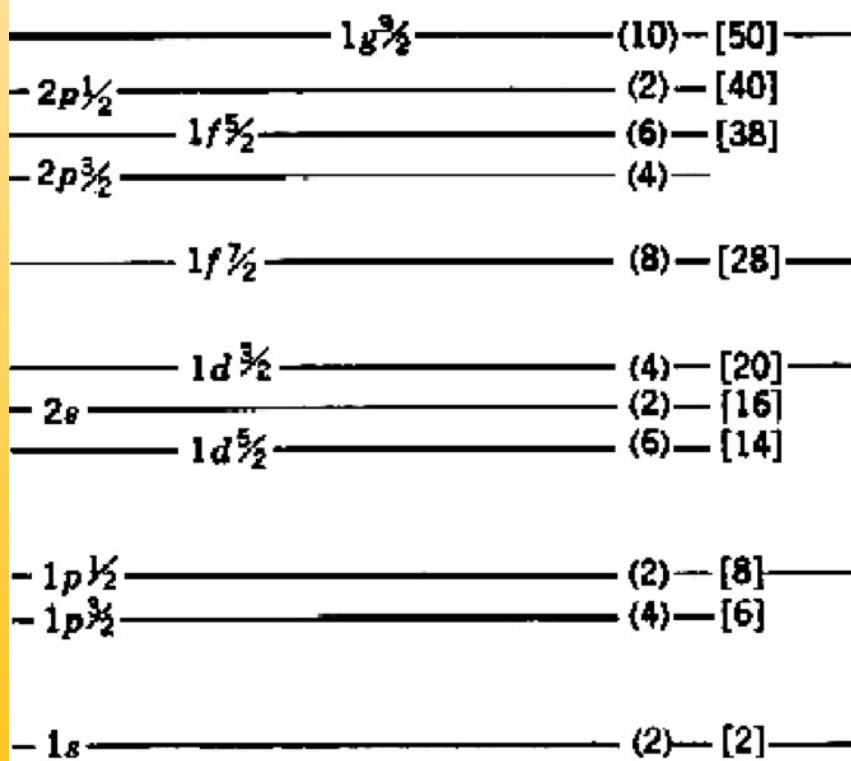


# Estudio del 170

**Extracción de los  
niveles de partícula  
simple del espectro  
experimental**

# Espectro experimental

$^{17}\text{O}$

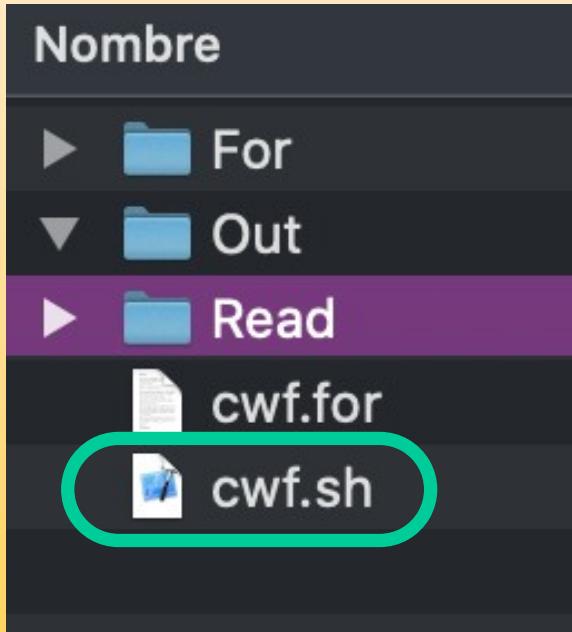


$^{16}\text{O}_8$

**Código**

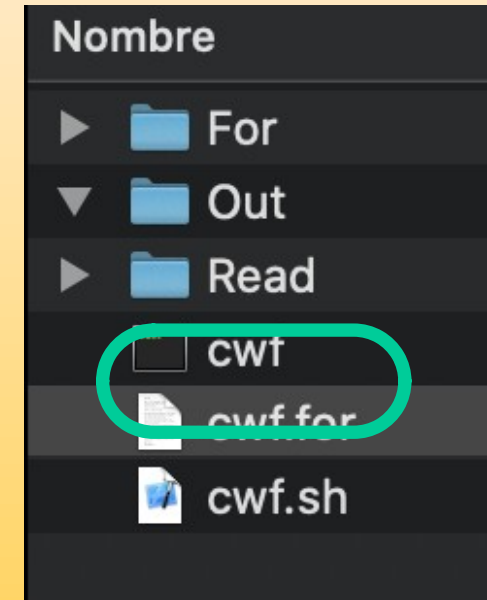
# Programa Numérico

## Compilar



```
Programa_Anti -  
[IB:Programa_Anti idbetan$ sh cwf.sh
```

```
Programa_Anti -  
Compiling cwf.for  
Please wait...  
... Done.  
IB:Programa_Anti idbetan$
```

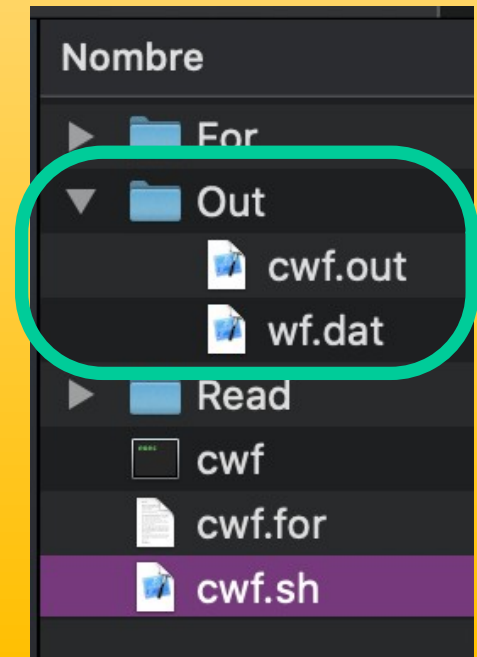


## Ejecutar

```
Programa_Anti -  
[IB:Programa_Anti idbetan$ ./cwf
```

```
Programa_Anti - -bash - 80x24  
[IB:Programa_Anti idbetan$ ./cwf  
1,2*j,e -> 3 7 0.2426669E+01 -0.1020101E+00  
IB:Programa_Anti idbetan$
```

## Output



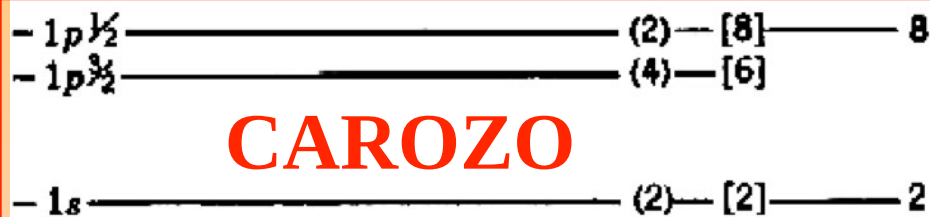
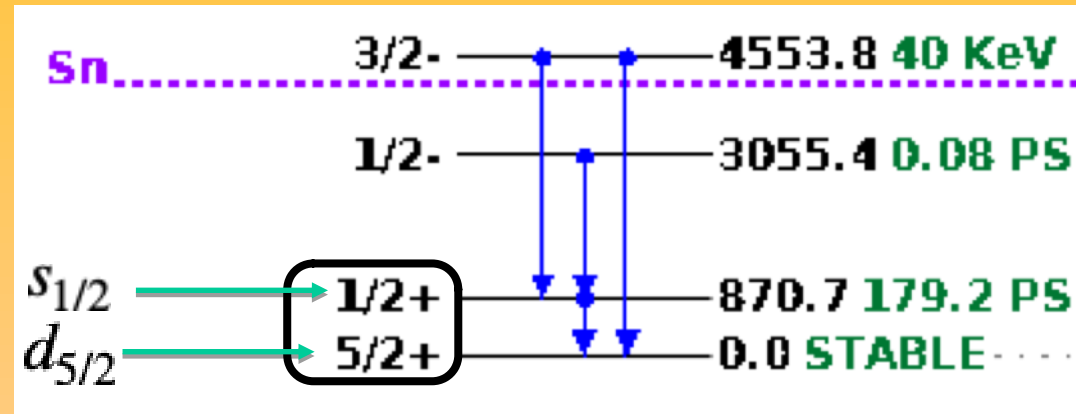
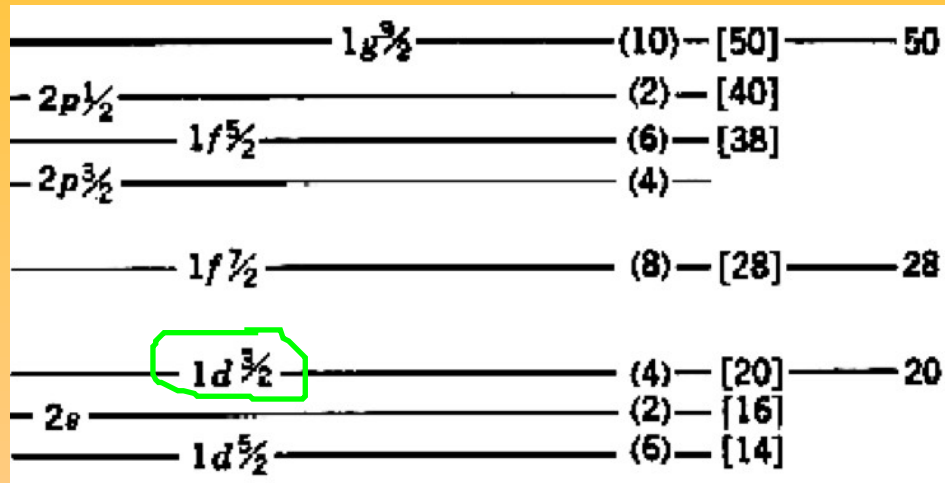


# Cálculo de los estados de partícula independiente del $^{17}\text{O}$

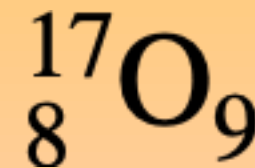


$$d_{5/2} \longrightarrow l = 2, j = 5/2$$

## VALENCIA



## CAROZO



Un neutrón en un núcleo mágico

Núcleo doblemente mágico

# Calculando ground state $^{17}\text{O}$

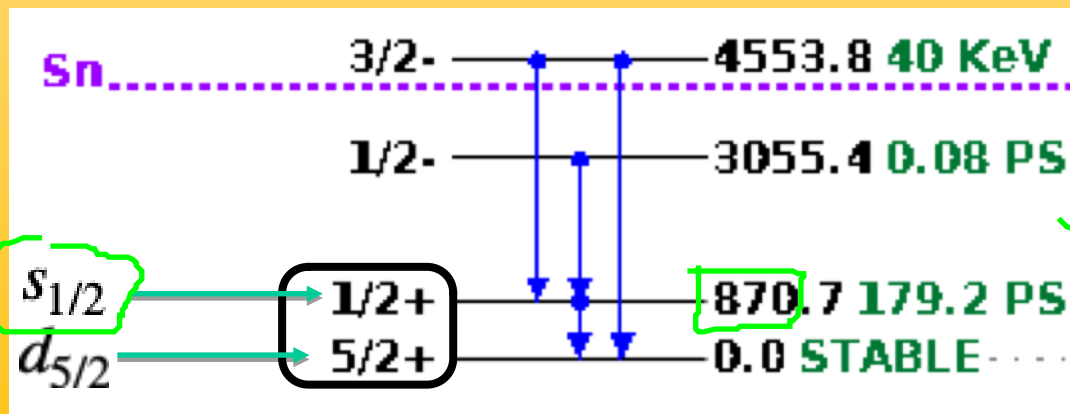
$$d_{5/2} \longrightarrow l = 2, j = 5/2$$

$$\epsilon_{d_{5/2}} = -S_n(^{17}\text{O})$$

$^{17}_8\text{O}$	
Major Modes	$S_n$ (keV)
	4143.0801



$$\epsilon_{d_{5/2}} = -4.143 \text{ MeV}$$



Handwritten green annotations:

- $s_{1/2}$  (circled) with an arrow pointing to the  $1/2^+$  level.
- $d_{5/2}$  (circled) with an arrow pointing to the  $5/2^+$  level.
- Vertical text  $5 \ 1/2$  and  $5 \ 2/2$  near the  $3/2^-$  and  $1/2^-$  levels.
- Horizontal lines with values  $-5.2$  and  $-4.1$  on the right side.

# ground state : input file <sup>17</sup>O

$$d_{5/2} \longrightarrow l = 2, j = 5/2$$

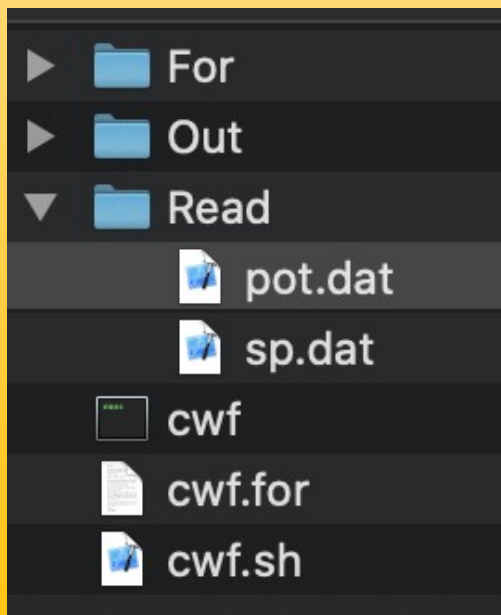
$$\epsilon_{d_{5/2}} = -4.143 \text{ MeV}$$

sp.dat

```
sp.dat / No Selection
-1 2 5 -4. 0. !itysp, 1, 2j, Re(E),Im(E)
itysp=-2 antibound states
itysp=-1 bound states
itysp=0 resonance states
itysp=1 scattering states
-----
```

pot.dat

```
20. 0.1
4.
1
16. 8. 44.1 1.25 0.65| 22.84
=====
COMMENT: 170=160+n
=====
rmax,rstep !rmax and rstep are the max radius and the step at which
           u(r) is evaluatedc
rmatch !rmatch is the matching radius
icore !icore=1(2) for neutrons(protons)
at zt v_0 r_0 a v_so !Woods-Saxon parameters
```



# ground state : Función de onda

$^{17}\text{O}$

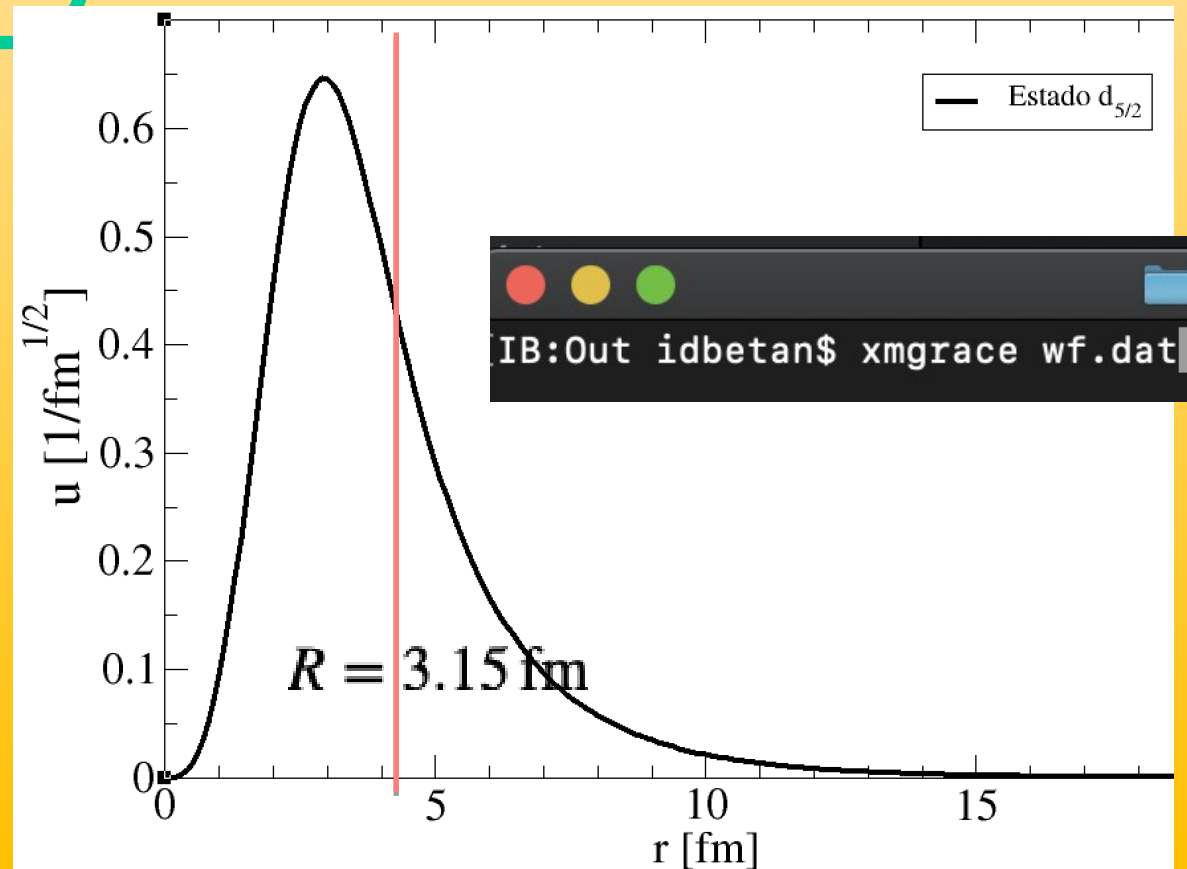
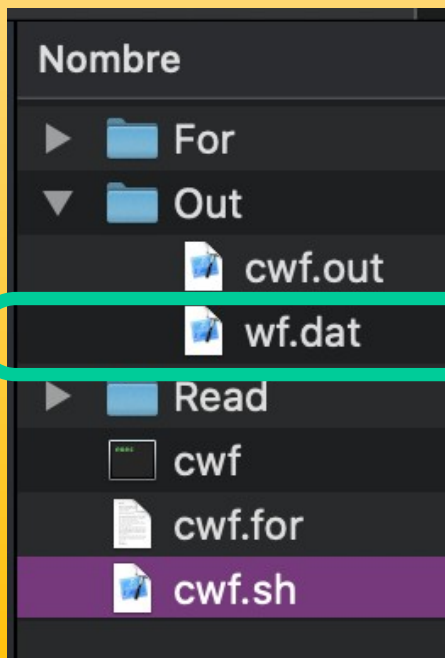
$$d_{5/2} \longrightarrow l = 2, j = 5/2$$

$$\varepsilon_{d_{5/2}}^{cal} = -4.14807 \text{ MeV}$$

$$V_0 = 53.202 \text{ MeV}$$

$$V_{so} = 11.068 \text{ MeV}$$

$$\varepsilon_{d_{5/2}} = -4.143 \text{ MeV}$$



**Estado excitado**

$^{17}\text{O}$

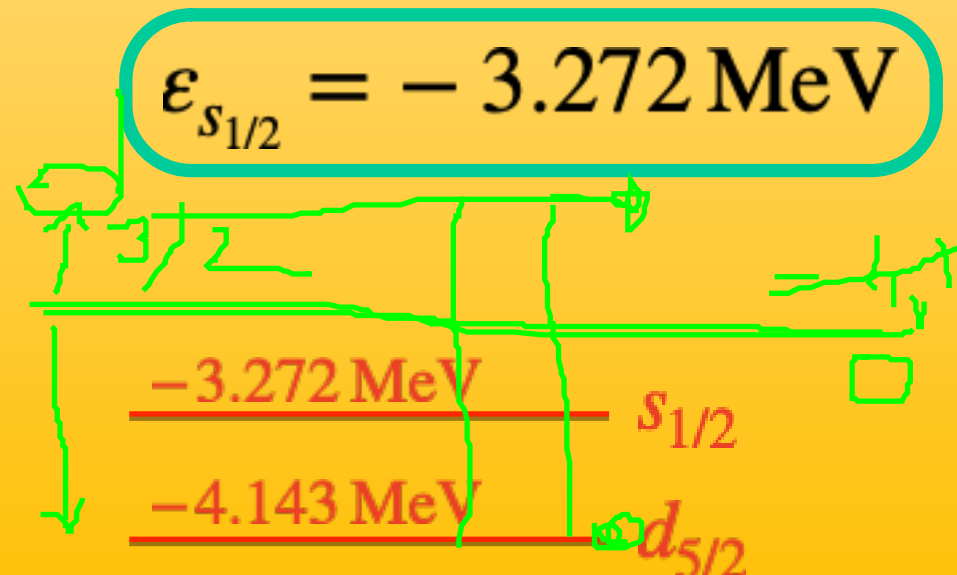
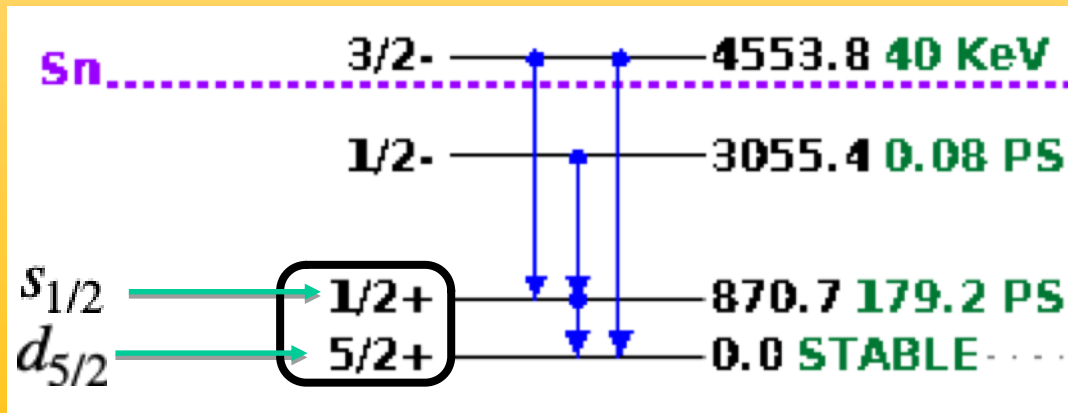
# 1er estado excitado del $^{17}\text{O}$

$$s_{1/2} \longrightarrow l = 0, j = 1/2$$

$^{17}_8\text{O}$	
γ Modes	$S_n$ (keV)
	4143.0801



$$\epsilon_{s_{1/2}} = (0.871 - 4.143) \text{ MeV}$$



# $^{17}\text{O}$ 1er estado excitado del

$$s_{1/2} \longrightarrow l = 0, j = 1/2$$

$$\epsilon_{s_{1/2}} = -3.272 \text{ MeV}$$

sp.dat

```
> sp.dat > No Selection
-1 0 1 -2| 0. !itysp, 1, 2j, Re(E),Im(E)
itysp=2 antibound states
```

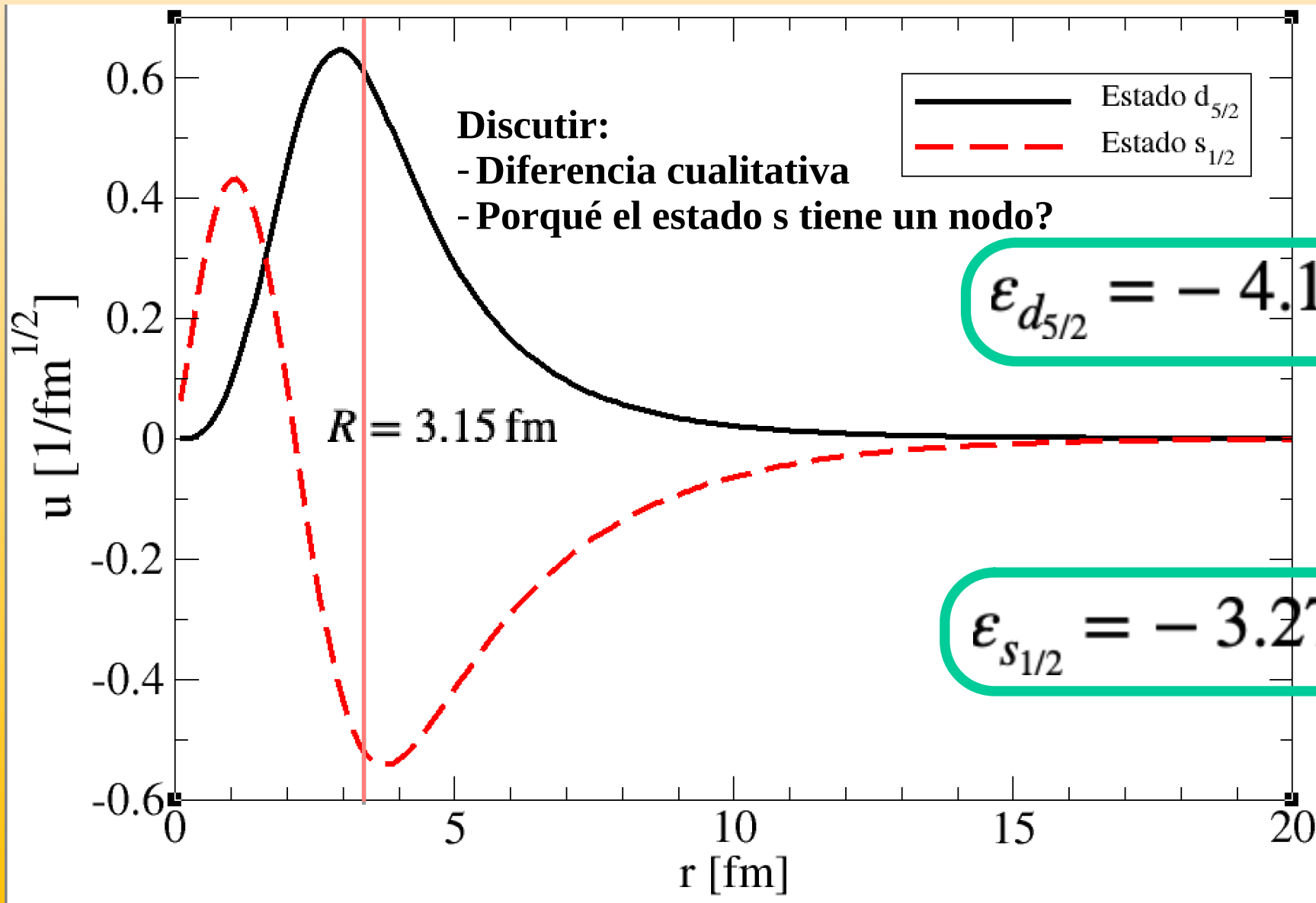
pot.dat

```
20. 0.1
4.
1
16. 8. 44.1 1.25 0.65| 22.84
=====
COMMENT: 170=160+n
=====
rmax,rstep !rmax and rstep are the max radius and the step at which
u(r) is evaluatedc
rmatch !rmatch is the matching radius
icore !icore=1(2) for neutrons(protons)
at zt v_0 r_0 a v_so !Woods-Saxon parameters
```

Reajustar??? Cuál???

$$\epsilon_{s_{1/2}}^{cal} = -3.27648 \text{ MeV}$$

# <sup>17</sup>C Compara estado fundamental y 1er excitado del





**Root-mean-square  
radius**

# Actividades Propuestas

## **Repetir para un protón en el mismo carozo:**

- Identificar los niveles de partícula simple
- Identificar la separation energy y asociarlo al estado fundamental
- Dar los niveles con respecto al carozo
- Graficar las funciones de onda
- Comparar la localización de las funciones de onda del estado fundamental y excitados
- Discutir sobre el número de nodos de los estados fundamental y excitados

## **Otros cálculos:**

- Calcular sin spin orbit y ver los cambios: analizar la degeneración de los estados
- Volver a encender el spin orbit y ver que cambio para los mismos estados.  
Estudiar el apartamiento de cada estado.

**Fin**