## Introducción a la Física Nuclear 2024

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# **Campo Medio**

#### **Contenido:**

- Campo medio y fuerza residual
- Potencial de Hartree-Fock
- Separabilidad de la función de onda
- Oscilador armónico; Woods-Saxon
- Interacción espín-momento orbital
- Núcleos mágicos

#### Lecturas sugeridas para Campo Medio:

- Cap. 3 libro J. Suhonen. From nucleons to nucleus. Springer-Verlag Berlin. 2007
- Cap. 7 libro S. S. M. Wong. Introductory Nuclear Physics. Willey. 2004

# Modelo de capas analítico

# Aproximación de campo medio

Hamiltoniano de muchos cuerpos

$$egin{array}{rcl} egin{array}{rcl} &=& T+V \ &=& \displaystyle{\sum_{i=1}^{A} \left[-rac{\hbar^2}{2m_i}
ight] 
abla^2_{m{r}_i} + \displaystyle{\sum_{i< j=1}^{A} v(m{r}_i,m{r}_j)} \end{array}} \end{array}$$



#### Hamiltoniano de muchos cuerpos

Modelo sin interacción

Hamiltoniano de<br/>muchos cuerpos $H_0$ independientes $H_0$ 

#### Hamiltoniano de muchos cuerpos

Modelo con interacción
$$H = H_0 + V_{res}$$

# **Campo medio**

Hamiltoniano de muchos cuerpos independientes  $H_0 = \sum_{i=1}^{n} h(\boldsymbol{r}_i)$ Modelo de capas  $\begin{array}{ccc} 1d_{3/2} & (4) \\ 2s_{1/2} & (2) \\ 1d_{5/2} & (6) \\ 0g_{7/2} & (8) \end{array}$ (50) 2s 1d 0g Hamiltoniano de partícula simple P=+  $0g_{0/2}$  (10)  $\begin{array}{ccc} 1p_{1/2} & (2) \\ 1p_{3/2} & (4) \\ 0f_{5/2} & (6) \end{array}$ 1p 0f (28)  $h(oldsymbol{r}) = \left[-rac{\hbar^2}{2m}
ight]
abla_{oldsymbol{r}}^2 + v(oldsymbol{r})$ 0f<sub>7/2</sub> (8)  $\begin{array}{ccc} 1s_{1/2} & (2) \\ 0d_{3/2} & (4) \\ 0d_{5/2} & (6) \end{array}$ 2 -0p  $\begin{array}{ccc} 0p_{1/2} & (2) \\ 0p_{3/2} & (4) \end{array}$ P=-(2)0s<sub>1/2</sub> (2) Campo medio  $v(\boldsymbol{r})$  $WS + v_{SO}$ WS Fuerzas: volumen + centrífuga + spin-orbit Oscilador Woods-Woods-Saxor harmónico Saxon +Spin-orbit

# Ecuación de Schroedinger sin spin-orbit

$$h(\boldsymbol{r}) = \frac{1}{2m}\boldsymbol{p}^2 + v(r) \qquad \boldsymbol{p}^2 = -\hbar^2 \nabla_{\boldsymbol{r}}^2$$

$$h(\mathbf{r}) = -\frac{\hbar^2}{2m} \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) - \frac{1}{r^2} \overline{l}^2 \right] + v(r)$$

#### **Momento angular orbital**

Aquí definido adimensional!!

$$v(r) = v_{nuclear}(r) + v_{coul}(r) \qquad \overline{l}^2 = -\left[\frac{1}{sen\theta}\frac{\partial}{\partial\theta}\left(sen\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{sen^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$$
$$v_{coul}(r) = \frac{Ze^2}{4\pi\epsilon_0} \begin{cases} \frac{1}{2R}\left(3 - \frac{r^2}{R^2}\right) & r \le R\\ \frac{1}{r} & r > R \end{cases}$$

iacei las cueillas...

Notación espectroscópica

$$l=0, 1, 2, 3, 4, 5, \dots$$
  
s, p, d, f, ,g ,h ...

$$\psi_{nsm_slm_l}(r,\theta,\phi) = \frac{1}{r} R_{nl}(r) \chi_{sm_s} Y_{lm_l}(\theta,\phi)$$

$$\frac{\hbar^2}{2m} \frac{d^2 R_{nl}(r)}{dr^2} + \left[\varepsilon_{nl} - v(r) - \frac{\hbar^2}{2m} \frac{l(l+1)}{r^2}\right] R_{nl}(r) = 0$$

## Funciones de onda para potencial sin spin-orbit

$$\psi_{nsm_slm_l}(r,\theta,\phi) = \frac{1}{r} R_{nl}(r) \chi_{sm_s} Y_{lm_l}(\theta,\phi)$$

$$h(\mathbf{r}) = -\frac{\hbar^2}{2m} \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) - \frac{1}{r^2} \bar{l}^2 \right] + v(r)$$

$$\frac{\hbar^2}{2m} \frac{d^2 R_{nl}(r)}{dr^2} + \left[\varepsilon_{nl} - v(r) - \frac{\hbar^2}{2m} \frac{l(l+1)}{r^2}\right] R_{nl}(r) = 0$$

Hacer las cuentas...

En paralelo hacer las cuentas con spin-orbit... **Solución analítica :** - W.S. con l=0

- U O
- H.O.
- Cosh

## Funciones de onda para potencial sin spin-orbit

$$\psi_{nsm_slm_l}(r, heta,\phi) = rac{1}{r} \, R_{nl}(r) \, \chi_{sm_s} \, Y_{lm_l}( heta,\phi)$$

#### Solución numérica

 $\frac{\hbar^2}{2m}\frac{d^2R_{nl}(r)}{dr^2} + \left[\varepsilon_{nl} - v(r) - \frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}\right]R_{nl}(r) = 0 \qquad \qquad \begin{array}{c} l = 0, \ 1, \ 2, \ 3, \ 4, \ 5, \ \dots \\ s, \ p, \ d, \ f, \ ,g \ ,h \ \dots \end{array}$ 

Convención  $\rightarrow n_{inicial} = 1$ 1s,1p,1d,...,2s,2p,2d,...

 $0s, 0p, 0d, \dots, 1s, 1p, 1d, \dots$ 

#### **Ejemplos**



Credit: A. deShalit and I. Talmi. Nuclear Shell Theory. 1963.

# Construcción de campo medio autoconsistente: Hartree-Fock

Función de onda de A cuerpos

 $\Psi(1,2,\cdots,A) = \mathcal{A}\psi_{\alpha_1}(r_1)\psi_{\alpha_2}(r_2)\cdots\psi_{\alpha_A}(r_A)$ 

Determinante de Slatter

**Principio variacional**  $\delta \langle \Psi | H | \Psi \rangle = \langle \delta \Psi | H | \Psi \rangle = 0$ 

#### Hamiltoniano de A cuerpos

$$H = \sum_{i=1}^{A} \left[ -rac{\hbar^2}{2m_i} 
ight] 
abla^2_{m{r}_i} + rac{1}{2} \sum_{i < j=1}^{A} v(m{r}_i, m{r}_j)$$

$$\begin{array}{ll} \begin{array}{ll} \displaystyle \begin{array}{c} \displaystyle \begin{array}{c} \displaystyle \begin{array}{c} \displaystyle \operatorname{Hamiltoniano} \ de \ un \ cuerpo} \\ \displaystyle \begin{array}{c} \displaystyle -\frac{\hbar^2}{2m} \nabla^2 \psi_{\alpha_i}(\boldsymbol{r}) & + \end{array} & \displaystyle \begin{array}{c} \displaystyle \sum_{j=1}^A \int d\boldsymbol{r}' \ \psi^*_{\alpha_j}(\boldsymbol{r}') \ v(\boldsymbol{r}, \boldsymbol{r}') \ \psi_{\alpha_j}(\boldsymbol{r}') \ \psi_{\alpha_i}(\boldsymbol{r}) \\ \\ \displaystyle \begin{array}{c} \displaystyle \begin{array}{c} \displaystyle \operatorname{Lo} \ vamos \ a \\ \displaystyle \begin{array}{c} \displaystyle \operatorname{demostrar} \ cuando \\ \displaystyle \operatorname{hagamos} \ segunda \\ \displaystyle \operatorname{cuantización} \end{array} & \begin{array}{c} \displaystyle - \displaystyle \sum_{j=1}^A \int d\boldsymbol{r}' \ \psi^*_{\alpha_j}(\boldsymbol{r}') \ v(\boldsymbol{r}, \boldsymbol{r}') \ \psi_{\alpha_j}(\boldsymbol{r}) \ \psi_{\alpha_i}(\boldsymbol{r}) \\ \end{array} \\ \end{array} \\ \end{array} \\ \end{array}$$

# Ecuación de Hartree-Fock

$$-\frac{\hbar^2}{2m}\nabla^2\psi_{\alpha_i}(\mathbf{r}) + \sum_{j=1}^{A}\int d\mathbf{r}'\psi_{\alpha_j}^*(\mathbf{r}')\,v(\mathbf{r},\mathbf{r}')\,\psi_{\alpha_j}(\mathbf{r}')\,\psi_{\alpha_i}(\mathbf{r})$$

$$-\sum_{j=1}^{A}\int d\mathbf{r}'\psi_{\alpha_j}^*(\mathbf{r}')\,v(\mathbf{r},\mathbf{r}')\,\psi_{\alpha_j}(\mathbf{r})\,\psi_{\alpha_i}(\mathbf{r}') = \varepsilon_{\alpha_i}\psi_{\alpha_i}(\mathbf{r})$$

$$U(\mathbf{r},\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}')\sum_{j=1}^{A}\int d\mathbf{r}''\,v(\mathbf{r},\mathbf{r}'')\,\psi_{\alpha_j}(\mathbf{r}'')\,\psi_{\alpha_j}(\mathbf{r}'') - \sum_{j=1}^{A}\,v(\mathbf{r},\mathbf{r}')\,\psi_{\alpha_j}(\mathbf{r}')\,\psi_{\alpha_j}(\mathbf{r}')$$

$$-\frac{\hbar^2}{2m}\nabla_{\mathbf{r}}^2\psi_{\alpha_i}(\mathbf{r}) + \int d\mathbf{r}'U(\mathbf{r},\mathbf{r}')\psi_{\alpha_i}(\mathbf{r}') = \varepsilon_{\alpha_i}\psi_{\alpha_i}(\mathbf{r})$$
Ecuación de Schroedinger no local h(\mathbf{r})\psi(\mathbf{r}) = \varepsilon\psi(\mathbf{r})

# **Oscilador armónico**

# Oscilador armónico

# Tres dimensionesEcuación radial $h(r) \varphi(r) = \varepsilon \varphi(r)$ $h(r) R_{nl}(r) = \varepsilon_{nl} R_{nl}(r)$ $h(r) = -\frac{\hbar^2}{2m} \nabla^2 + V(r)$ $h(r) = -\frac{\hbar^2}{2m} \left[ \nabla_r^2 - \frac{l(l+1)}{r^2} \right] + \frac{1}{2} m \omega^2 r^2$ Potencial $V(r) = \frac{1}{2} m \omega^2 r^2$ $R_{nl}(r) = \sqrt{\frac{2n!}{b^3 \Gamma(n+l+\frac{3}{2})}} \left( \frac{r}{b} \right)^l e^{-\frac{r^2}{2b^2}} L_n^{l+\frac{1}{2}}(r^2/b^2)$

#### Autofunciones

 $\varphi_{nlm}(\boldsymbol{r}) = R_{nl}(r) Y_{lm}(\theta, \phi)$ 

$$\int_0^\infty r^2 dr R_{nl}(r) R_{n'l}(r) = \delta_{nn'}$$

# FrecuenciaAnchoPotencial $\hbar\omega = \frac{41}{A^{1/3}} MeV$ $b = \sqrt{\frac{\hbar}{m\omega}} = -\frac{1}{2}m\omega^2 r^2 = \frac{1}{2}\hbar\omega \left(\frac{r}{b}\right)^2$

## Oscilador armónico: autovalores

#### Autovalores

$$\varepsilon_N = \left(N + \frac{3}{2}\right) \hbar \omega \qquad N = 0, 1, 2, \cdots$$

$$\varepsilon_{nl} = \left(2n+l+rac{3}{2}
ight)\hbar\omega$$
 / permitidos  
 $l=N,N-2,\cdots,1, \text{ or } 0.$ 

#### Máximo estados en cada capa N

$$D_N = 2 \sum_{\text{allow } l} (2l+1) = 2 \sum_{k=1}^{N+1} k = (N+1)(N+2)$$



Credit: Fig. 3.5 of Ref. [22]

#### Máximo estados hasta la capa N<sub>max</sub>

$$D_{max} = \sum_{N=0}^{N_{max}} D_N = \frac{1}{3} (N_{max} + 1)(N_{max} + 2)(N_{max} + 3)$$
 (Acumulados)

# Oscilador armónico: números mágicos

$$h(r) = -\frac{\hbar^2}{2m} \left[ \nabla_r^2 - \frac{l(l+1)}{r^2} \right] + \frac{1}{2} m \omega^2 r^2 \qquad \hbar \omega = \frac{41}{A^{1/3}}$$

D

$$h(r) R_{nl}(r) = \varepsilon_{nl} R_{nl}(r)$$

$$\varepsilon_N = \left(N + \frac{3}{2}\right) \hbar \omega$$

$$N = 2n + l$$
$$N = 0, 1, 2, \cdots$$

$$max = \sum_{N=0}^{N_{max}} D_N = \frac{1}{3} (N_{max} + 1)(N_{max} + 2)(N_{max} + 3)$$
 (Acumulados)

# Oscilador armónico: números mágicos

$$h(r) = -\frac{\hbar^2}{2m} \left[ \nabla_r^2 - \frac{l(l+1)}{r^2} \right] + \frac{1}{2}m\omega^2 r^2$$

$$h(r) R_{nl}(r) = \varepsilon_{nl} R_{nl}(r)$$

$$\varepsilon_{N} = \left( N \pm \frac{3}{2} \right) \hbar \omega$$

$$\hbar\omega = \frac{41}{A^{1/3}}$$

$$\varepsilon_N = \left(N + \frac{3}{2}\right) \hbar \omega$$

 $N = 0, 1, 2, \cdots$ 

N = 2n+l

$$\varepsilon_{nl} = \left(2n+l+\frac{3}{2}\right)\hbar\omega$$

$$n = 0, 1, 2, \cdots$$
  
 $l = 0, 1, 2, \cdots$ 



Credit: Fig. 3.5 of Ref. [22]

# Oscilador armónico: números mágicos





Credit: Fig. 3.5 of Ref. [22]

# Woods-Saxon

#### Hamiltoniano de partícula individual: Pozo finito

$$h(\boldsymbol{r}) = \left[-rac{\hbar^2}{2m}
ight] 
abla^2_{\boldsymbol{r}} + v(\boldsymbol{r})$$

Woods-Saxon

$$\frac{\hbar^2}{2m} \frac{d^2}{dr^2} u_{nl}(r) + \left[ \varepsilon_{nl} - v_{WS}(r) - \frac{\hbar^2}{2m} \frac{l(l+1)}{r^2} \right] u_{nl}(r) = 0$$

#### Sobre los parámetros del WS:

$$v_{WS}(r) = \frac{-V_0}{1 + e^{\frac{r-R}{a}}}$$

 $R = r_0 A^{1/3}$ (+): protón, (-):neutrón  $V_0 = \left(51 \pm 33 \frac{N-Z}{A}\right) M e V$ 



# Espectro de energía del potencial de Woods-Saxon

Woods-Saxon

$$\frac{\hbar^2}{2m} \frac{d^2}{dr^2} u_{nl}(r) + \left[ \varepsilon_{nl} - v_{WS}(r) - \frac{\hbar^2}{2m} \frac{l(l+1)}{r^2} \right] u_{nl}(r) = 0$$

Números mágicos WS: 2, 8, 20, 40, 58

Números experimental: 2, 8, 20, 28, 50, 86



# Comparación pozos Infinito y finito: forma

#### Oscilador harmónico

$$\frac{\hbar^2}{2m} \frac{d^2}{dr^2} u_N(r) + \left[ \varepsilon_N - v_{HO}(r) - \frac{\hbar^2}{2m} \frac{l(l+1)}{r^2} \right] u_N(r) = 0$$

#### Woods-Saxon

$$\frac{\hbar^2}{2m} \frac{d^2}{dr^2} u_{nl}(r) + \left[ \varepsilon_{nl} - v_{WS}(r) - \frac{\hbar^2}{2m} \frac{l(l+1)}{r^2} \right] u_{nl}(r) = 0$$

$$v_{HO}(r) = \frac{1}{2}m\omega^2 r^2 \qquad \hbar\omega = \frac{41}{A^{1/3}}$$

$$v_{HO}(r) = \frac{1}{2}m\omega^2 r^2 = \frac{1}{2}m\frac{(\omega\hbar)^2}{\hbar^2}r^2 = \frac{1}{2}\frac{mc^2}{(\hbar c)^2}(\omega\hbar)^2 r^2$$



#### Comparación pozos Infinito y finito: funciones de onda



Comparación pozos Infinito y finito: espectro  $h(\mathbf{r}) = \left[-\frac{\hbar^2}{2m}\right] \nabla_{\mathbf{r}}^2 + v(\mathbf{r})$ 

Oscilador harmónico  

$$\frac{\hbar^2}{2m} \frac{d^2}{dr^2} u_N(r) + \left[\varepsilon_N - v_{HO}(r) - \frac{\hbar^2}{2m} \frac{l(l+1)}{r^2}\right] u_N(r) = 0$$

#### **Woods-Saxon**

$$\frac{\hbar^2}{2m} \frac{d^2}{dr^2} u_{nl}(r) + \left[ \varepsilon_{nl} - v_{WS}(r) - \frac{\hbar^2}{2m} \frac{l(l+1)}{r^2} \right] u_{nl}(r) = 0$$

Números mágicos del o.h.: 2, 8, 20, 40, 70 Números mágicos WS: 2, 8, 20, 40, 58 Números experimental: 2, 8, 20, 28, 50, 82



Credit: Fig. 3.5 of Ref. [22]

# Woods-Saxon con spin-orbit

$$h(\mathbf{r}) = -\frac{\hbar^2}{2m} \nabla^2 + V(r) + V_{so}(r) \mathbf{l} \cdot \mathbf{s}$$

$$\nabla^2 = \nabla_r^2 - \frac{\mathbf{l}^2/\hbar^2}{r^2} \qquad \nabla_r^2 = \frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d}{dr} \right)$$

$$h(\mathbf{r}) \psi_{nljm}(\mathbf{r}) = \varepsilon \psi_{nljm}(\mathbf{r})$$

$$\langle \boldsymbol{r} | \psi_{nljm} \rangle = \langle r | nlj \rangle \langle \hat{r} | lsjm \rangle$$

$$h(\mathbf{r}) = -\frac{\hbar^2}{2m} \nabla^2 + V(r) + V_{so}(r) \mathbf{l} \cdot \mathbf{s} \qquad \nabla^2 = \nabla_r^2 - \frac{\mathbf{l}^2/\hbar^2}{r^2}$$
$$\mathbf{j} = \mathbf{l} + \mathbf{s}$$

 $h(\mathbf{r}) \psi_{nljm}(\mathbf{r}) = \varepsilon \psi_{nljm}(\mathbf{r}) \qquad \langle \mathbf{r} | \psi_{nljm} \rangle = \langle r | nlj \rangle \langle \hat{r} | lsjm \rangle$ 

 $\mathbf{j}^2 |lsjm\rangle = j(j+1) |lsjm\rangle$ 

$$j_{z} | lsjm \rangle = m | lsjm \rangle$$
$$l^{2} | lsjm \rangle = l(l+1) | lsjm \rangle$$
$$s^{2} | lsjm \rangle = \frac{3}{4} | lsjm \rangle$$

$$\frac{\hbar^2}{2m} \frac{d^2}{dr^2} u_{nlj}(r) + \left[ \varepsilon_{nlj} - V(r) - \zeta_{l,j} V_{so}(r) - \frac{\hbar^2}{2m} \frac{l(l+1)}{r^2} \right] u_{nlj}(r) = 0$$
  
$$\zeta_{l,j=l+1/2} = l \qquad \zeta_{l,j=l-1/2} = -(l+1)$$



Números WS+so. : **2**, **8**, **20**, **28**, **50**, **82**, **126** Números experimental. : 2, 8, 20, 28, 50, 82, 126

# Modelo de partícula independiente

## Modelo de partícula independiente

$$H = \sum_{i=1}^{A} \left[ \frac{-\hbar^2}{2m_i} \right] \nabla_{r_i}^2 + \sum_{i< j=1}^{A} v(r_i, r_j) = H_0 + V$$

$$H = H_0 + V$$

$$H_0 = \sum_{i=1}^{A} h(r_i) \qquad h(r)\psi_\alpha(r) = \varepsilon_\alpha \psi_\alpha(r)$$

$$\Psi_{0}(r_{1}, \dots, r_{A}) = \psi_{\alpha_{1}}(r_{1}) \cdots \psi_{\alpha_{A}}(r_{A})$$

$$E_{0} = \sum_{i=1}^{A} \varepsilon_{\alpha_{i}}$$

$$H_{0}\Psi_{0} = E_{0}\Psi_{0}$$

# Definición de carozo y valencia



Crédito: Fig. 4.1 J. Suhonen. From Nucleons to Nucleus 2007

# Definición de Núcleos Mágicos



# Modelo de capa para capas cerradas

# Estado fundamental 0+ de núcleos par-par



 $\Box_{20}^{40} Ca_{20}$ 

# Modelo de capa para un neutrón de valencia

# Números cuánticos versus estados experimentales

# **Modelo:** $\Box_{9}^{17}O_{8}\overset{!}{}_{8}^{16}O_{8}+n$



-1012

-10<sup>3</sup>





 $\Box_{8}^{17}O_{q}$ 

#### Experimento

#### Energía del Estado Fundamental: neutrón de valencia



# Modelo de capa para un protón de valencia

# Números cuánticos versus estados experimentales

# **Modelo:** $\Box_{9}^{17}F_{8}\overset{!}{}_{8}^{16}O_{8}+p$

**Estados excitados** 



-1012

-1n≫





#### Experimento

#### Energía del Estado Fundamental: protón de valencia

$$S_p({}_{9}^{1/}F_8) = B(Z = 9, N = 8) - B(Z - 1 = 8, N = 8)$$

 $S_p = 0.600 \,\mathrm{MeV}$ 



# Modelo de capa para un neutrón y un protón de valencia



