Introducción a la Física Nuclear 2023

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Álgebra de momentos angulares

Contenido:

Definición de momento angular. Deducción de las matrices de Pauli. Acople de dos momentos angulares. Coeficientes de Clebsch-Gordan. Ejemplo de acoples. Acople de tres y cuatro momentos angulares. Cambio de acoples y símbolo de Wigner 6j y 9j. Ejemplo de acople de cuatro momentos angulares.

Lectura recomendada:

Capítulo 1 del libro From Nucleons to Nucleus. Concepts of Microscopic Nuclear Theory. J. Suhonen. Springer. 2007.

Motivaciones:

- Momento angular total de una partícula con spin: acople *ls*

- Momento angular de dos partículas: acople *jj* o *LS*

- Momento angular de isospin: acople de dos nucleones

Motivación para el uso de momentos angulares acoplados

Función de onda de una partícula

$\phi_{nlj}(r)$ $Y_{lm_l}(\theta,\phi)$ χ_{sm_s}

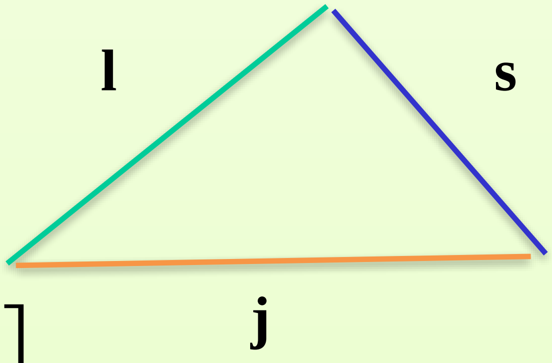
s: momento intrínseco de espín **l:** momento angular orbital

l y s desacoplados

$$\psi_{nljm}(r,s)=\phi_{nlj}(r)\chi_{sm_s}Y_{lm_l}(\theta,\phi)$$

Partícula con spin

l y s acoplados



$$\psi_{nljm}(r,s)=\phi_{nlj}(r)\left[\chi_s Y_l(\theta,\phi)\right]_{jm}$$

Coeficientes de Clebsch-Gordan

$$\left[\chi_s Y_l(\theta,\phi)\right]_{jm}=\sum_{m_s,m_l}\langle s\, m_s\, l\, m_l | j\, m\rangle \chi_{s m_s} Y_{l m_l}(\theta,\phi)$$

Función de onda de dos partículas

Contexto: Sistema Many-Body Finito

Función de onda de dos partículas en acople sl
(ver aplicaciones al final)

$$\langle \boldsymbol{x}_1 \boldsymbol{x}_2 | l_a l_b SL, JM \rangle \quad = \quad \phi_a(r_1)\, \phi_b(r_2) \left[[\chi_{s_1}(1) \chi_{s_2}(2)]_S \, [Y_{l_a}(\hat{r}_1) Y_{l_b}(\hat{r}_2)]_L \right]_{JM}$$

Función de onda de dos partículas en acople jj
(ver aplicaciones al final)

$$\langle \boldsymbol{x}_1 \boldsymbol{x}_2 | j_a j_b, JM \rangle \quad = \quad \phi_a(r_1)\, \phi_b(r_2) \left[[\chi_{s_1}(1)\, Y_{l_a}(\hat{r}_1)]_{j_a} \, [\chi_{s_2}(2)\, Y_{l_b}(\hat{r}_2)]_{j_b} \right]_{JM}$$

Definición de momento angular

Definición

$$J = (J_1, J_2, J_3)$$

$$J^\dagger = J$$

Relaciones de conmutación

$$[J_1, J_2] = J_1 J_2 - J_2 J_1 = i\hbar J_3$$
$$[J_2, J_3] = i\hbar J_1$$
$$[J_3, J_1] = i\hbar J_2$$

Verificar que tiene unidades de impulso angular

$$[J_i, J_j] = i\hbar \sum_k \epsilon_{ijk} J_k$$

Coeficientes de Levi-Civita

$$\epsilon_{123} = \epsilon_{231} = \epsilon_{312} = 1$$

$$\epsilon_{132} = \epsilon_{321} = \epsilon_{213} = -1$$

$$\epsilon_{121} = \epsilon_{112} = \epsilon_{212} = \ldots = 0$$

Ejemplo

$$[J_1, J_2] = i\hbar \sum_{k=1}^{3} \epsilon_{12k} J_k \qquad [J_1, J_2] = i\hbar J_3$$

Autovectores

Operadores

$$J = (J_x, J_y, J_z)$$

$$J^2 = J_x^2 + J_y^2 + J_z^2$$

Autovectores

$$| jm \rangle$$

Autovalores de

$$J^2 | jm \rangle = \hbar^2 j(j+1) | jm \rangle$$

j: entero o semi-entero

Autovalores de

$$J_z | jm \rangle = \hbar m | jm \rangle$$

m=-j, -j+1,…, j-1, j

Base

$J = (J_x, J_y, J_z)$

Ortonormalidad

$$\langle jm | j'm' \rangle = \delta_{jj'} \delta_{mm'}$$

Autovectores

$$| jm \rangle$$

Completitud

$$I = \sum_{jm} | jm \rangle \langle jm |$$

Base

Coeficientes $\langle l m_l | f \rangle$

$$| f \rangle \longrightarrow | f \rangle = I | f \rangle$$

$$| f \rangle = \sum_{l m_l} | l m_l \rangle \langle l m_l | f \rangle = \sum_{l m_l} f_{lm} | l m_l \rangle$$

Armónicos esféricos

Ejemplo: Momento angular orbital

Representación abstracta

$$l = (l_x, l_y, l_z)$$

Autovectores

$$l^2 = l_x^2 + l_y^2 + l_z^2$$

$$l^2 || l\, m_l \rangle = \hbar^2 l(l+1) || l\, m_l \rangle$$

$$l_z || l\, m_l \rangle = \hbar m_l || l\, m_l \rangle$$

Autovalores

$$l = 0, 1, 2, \cdots$$

$$m_l = -l, -l+1, \cdots, 0, \cdots, l$$

Ortonormalidad

$$\langle l m | l' m' \rangle = \delta_{l l'} \delta_{m m'}$$

Completitud

$$I = \sum_{m=-l}^{l} \sum_{l=0}^{\infty} || l m \rangle \langle l m ||$$

Armónicos esféricos

Representación coordenadas

Armónicos esféricos (AE)

$$Y_{lm}(\theta,\phi)=\langle\theta\phi|lm\rangle \rightarrow ||lm\rangle\rangle$$

$$\rightarrow ||\theta\phi\rangle\rangle$$

$$\overline{Y}_{lm}(\theta,\phi)=\langle lm|\theta\phi\rangle$$

$$l^2 Y_{lm}(\theta,\phi)=\hbar^2 l(l+1) Y_{lm}(\theta,\phi)$$

$$l_z Y_{lm}(\theta,\phi)=\hbar m_l Y_{lm}(\theta,\phi)$$

Base para el espacio angular

$$\langle\theta\phi|\theta'\phi'\rangle=\frac{\delta(\theta-\theta')}{\sin\theta}\delta(\phi-\phi')$$

$$I=\int_0^{\pi}\sin\theta\, d\theta\int_0^{2\pi} d\phi\, ||\theta\phi\rangle\rangle\langle\langle\theta\phi||$$

Ejemplo: sgte. transparencia

Ortogonalidad de los armónicos esféricos

$$Y_{lm}(\theta,\phi)=\langle\theta\phi|lm\rangle$$

A partir de la ortogonalidad …

$$\overline{Y}_{lm}(\theta,\phi)=\langle lm|\theta\phi\rangle$$

$$\delta_{ll'}\delta_{mm'}=\langle lm|l'm'\rangle=\langle lm|I|l'm'\rangle$$

$$I=\int_0^{\pi}\sin\theta\,d\theta\int_0^{2\pi}d\phi\,||\theta\phi\rangle\langle\theta\phi||$$

$$.=\int_0^{\pi}\sin\theta\,d\theta\int_0^{2\pi}d\phi\,\langle lm|\theta\phi\rangle\langle\theta\phi|l'm'\rangle$$

$$.=\int_0^{\pi}\sin\theta\,d\theta\int_0^{2\pi}d\phi\,\overline{Y}_{lm}(\theta\phi)Y_{l'm'}(\theta\phi)=\delta_{ll'}\delta_{mm'}$$

Expansión en armónicos esféricos

Representación coordenadas

$$|lm_l\rangle \longrightarrow \langle\theta\phi|lm_l\rangle = Y_{lm_l}(\theta,\phi)$$

Expansión

$$|f\rangle = \sum_{lm_l} |lm_l\rangle\langle lm_l|f\rangle = \sum_{lm_l} f_{lm}|lm_l\rangle$$

$$\rightarrow \langle\theta\phi|f\rangle = \sum_{lm_l} f_{lm}\langle\theta\phi|lm_l\rangle = \sum_{lm_l} f_{lm} Y_{lm_l}(\theta,\phi)$$

$$\Rightarrow f(\theta,\phi) = \sum_{lm_l} f_{lm} Y_{lm_l}(\theta,\phi)$$

Notar que los coeficientes son los mismos

$$f_{lm} = \langle lm_l|f\rangle$$

Expresarlos como integral y ver que los coeficientes de Fourier

Operadores de crecimiento

Operador de crecimiento

$J = (J_1, J_2, J_3)$

$(\| jm\rangle)$

Raising and lowering operators

$$J_+ = J_1 + i\,J_2$$

$$J_- = J_1 - i\,J_2$$

Relación de conmutación

$[J_1, J_2] = i\hbar J_3$

$$[J_+, J_-] = 2\hbar J_3$$

Veamos… $[J_+, J_-] = [J_1 + i\,J_2, J_1 - i\,J_2] =$

$$= [J_1, J_1] - i[J_1, J_2] + i[J_2, J_1] - i^2[J_2, J_2] =$$

$$= 0 - i(i\hbar J_3) + i(-i\hbar J_3) - 0 =$$

$$= -2i^2\hbar J_3 = 2\hbar J_3$$

Operador de crecimiento

$$J = (J_1, J_2, J_3)$$

$$(\| jm \rangle)$$

Raising and lowering operators

$$J_+ = J_1 + i\,J_2$$

$$J_- = J_1 - i\,J_2$$

Relación de conmutación

$$[J_+, J_-] = 2\hbar J_3$$

$$[J_+, J_3] = -\hbar J_+$$

$$[J_-, J_3] = \hbar J_-$$

$$[J_+, \boldsymbol{J}^2] = 0$$

$$[J_-, \boldsymbol{J}^2] = 0$$

Asignado como TP

Como ejemplo de aplicación, vamos a usar estas identidades en la siguiente transparencia

Uso de los operadores de crecimiento

$\boldsymbol{J} = (J_x, J_y, J_z)$ $\boldsymbol{J}^2 \,|\, jm\rangle = \hbar^2 j(j+1) \,|\, jm\rangle$

$$J_+ = J_x + i\,J_y \qquad J_- = J_x - i\,J_y$$

Generación del autovector (j,m+1):

Veamos… consideremos el vector $J_+ \,|\, jm\rangle$ y veamos que es autovector de $\boldsymbol{J}^2$ y J_z

$$\boldsymbol{J}^2(J_+ \,|\, jm\rangle) = \hbar^2 j(j+1)(J_+ \,|\, jm\rangle) \longleftarrow [J_\pm, \boldsymbol{J}^2] = 0$$

$$J_z(J_+ \,|\, jm\rangle) = \hbar(m+1)(J_+ \,|\, jm\rangle) \longleftarrow [J_+, J_z] = -\hbar J_+$$

Propiedades de y

$$\boldsymbol{J} = (J_x, J_y, J_z) \qquad \boldsymbol{J}^2 \,|\, jm\rangle = \hbar^2 j(j+1) \,|\, jm\rangle$$

$$J_+ = J_x + i\,J_y \qquad J_- = J_x - i\,J_y$$

Generación del autovector (j,m+1):

Condon-Shortley phase convention assumed (ver más adelante)

$$J_+ \,|\, jm\rangle = \hbar\sqrt{j(j+1) - m(m+1)}\,|\, jm+1\rangle$$

$$J_- \,|\, jm\rangle = \hbar\sqrt{j(j+1) - m(m-1)}\,|\, jm-1\rangle$$

Aplicación de los operadores $J_{\pm}$: Matrices de Pauli

Deducción de las matrices de Pauli

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\sigma = (\sigma_x, \sigma_y, \sigma_z)$$

Espín

$$S = (S_x, S_y, S_z)$$

$$S = \frac{\hbar}{2}\sigma$$

Autovectores

$$S^2 ||s\, m_s\rangle = \hbar^2 s(s+1) ||s\, m_s\rangle$$

$$S_z ||s\, m_s\rangle = \hbar m_s ||s\, m_s\rangle$$

$$s = \frac{1}{2}$$

$$m_s = \frac{-1}{2}, \frac{1}{2}$$

Cálculo de la realización de S_x

$$S = \frac{\hbar}{2}\sigma \qquad S = (S_x, S_y, S_z)$$

Matrices de Pauli

$$\sigma = (\sigma_x, \sigma_y, \sigma_z)$$

$$S^2 \| s\, m_s \rangle = \frac{3}{4}\hbar^2 \| s\, m_s \rangle$$

$$S_z \| s\, m_s \rangle = \hbar m_s \| s\, m_s \rangle$$

$$\langle s\, m_s | s'\, m_s' \rangle = \delta_{s\,s'}\delta_{m_s m_s'}$$

Podemos omitir el primer factor ½ en todos bracket

$$S_x = \begin{pmatrix} \langle \frac{1}{2}, \frac{1}{2} | S_x | \frac{1}{2}, \frac{1}{2} \rangle & \langle \frac{1}{2}, \frac{1}{2} | S_x | \frac{1}{2}, -\frac{1}{2} \rangle \\ \langle \frac{1}{2}, -\frac{1}{2} | S_x | \frac{1}{2}, \frac{1}{2} \rangle & \langle \frac{1}{2}, -\frac{1}{2} | S_x | \frac{1}{2}, -\frac{1}{2} \rangle \end{pmatrix}$$

$$S_+ = S_x + i S_y$$
$$S_- = S_x - i S_y$$

$$\Rightarrow \quad S_x = \frac{S_+ + S_-}{2}$$

$$S_\pm | s, m_s \rangle = \hbar \sqrt{s(s+1) - m_s(m_s \pm 1)}\, | s, m_s \pm 1 \rangle$$

Cálculo de la realización de S_x

$$S=\frac{\hbar}{2}\sigma \qquad S=(S_x,S_y,S_z) \qquad \sigma=(\sigma_x,\sigma_y,\sigma_z)$$

$$S_x=\begin{pmatrix} \langle\frac{1}{2},\frac{1}{2}|S_x|\frac{1}{2},\frac{1}{2}\rangle & \langle\frac{1}{2},\frac{1}{2}|S_x|\frac{1}{2},-\frac{1}{2}\rangle \\ \langle\frac{1}{2},-\frac{1}{2}|S_x|\frac{1}{2},\frac{1}{2}\rangle & \langle\frac{1}{2},-\frac{1}{2}|S_x|\frac{1}{2},-\frac{1}{2}\rangle \end{pmatrix}=\frac{\hbar}{2}\sigma_x$$

$$S_x=\frac{S_+ + S_-}{2} \qquad S_\pm|s,m_s\rangle=\hbar\sqrt{s(s+1)-m_s(m_s\pm 1)}\,|s,m_s\pm 1\rangle$$

$$S_+\left|\frac{1}{2},\frac{1}{2}\right\rangle=0 \qquad S_-\left|\frac{1}{2},\frac{1}{2}\right\rangle=\hbar\left|\frac{1}{2},-\frac{1}{2}\right\rangle$$

$$S_+\left|\frac{1}{2},-\frac{1}{2}\right\rangle=\hbar\left|\frac{1}{2},\frac{1}{2}\right\rangle \qquad S_-\left|\frac{1}{2},-\frac{1}{2}\right\rangle=0$$

Expresiones de las Matrices S_+ *y* S_-.

$$S = \frac{\hbar}{2}\sigma \qquad S = (S_x, S_y, S_z) \qquad \sigma = (\sigma_x, \sigma_y, \sigma_z)$$

$$S_+ = \begin{pmatrix} \langle \frac{1}{2}, \frac{1}{2} | S_+ | \frac{1}{2}, \frac{1}{2} \rangle & \langle \frac{1}{2}, \frac{1}{2} | S_+ | \frac{1}{2}, -\frac{1}{2} \rangle \\ \langle \frac{1}{2}, -\frac{1}{2} | S_+ | \frac{1}{2}, \frac{1}{2} \rangle & \langle \frac{1}{2}, -\frac{1}{2} | S_+ | \frac{1}{2}, -\frac{1}{2} \rangle \end{pmatrix}$$

$$S_- = \begin{pmatrix} \langle \frac{1}{2}, \frac{1}{2} | S_- | \frac{1}{2}, \frac{1}{2} \rangle & \langle \frac{1}{2}, \frac{1}{2} | S_- | \frac{1}{2}, -\frac{1}{2} \rangle \\ \langle \frac{1}{2}, -\frac{1}{2} | S_- | \frac{1}{2}, \frac{1}{2} \rangle & \langle \frac{1}{2}, -\frac{1}{2} | S_- | \frac{1}{2}, -\frac{1}{2} \rangle \end{pmatrix}$$

$$\boxed{S_x = \frac{S_+ + S_-}{2}}$$

$$S_+ \left| \frac{1}{2}, \frac{1}{2} \right\rangle = 0 \qquad S_- \left| \frac{1}{2}, \frac{1}{2} \right\rangle = \hbar \left| \frac{1}{2}, -\frac{1}{2} \right\rangle$$

$$S_+ \left| \frac{1}{2}, -\frac{1}{2} \right\rangle = \hbar \left| \frac{1}{2}, \frac{1}{2} \right\rangle \qquad S_- \left| \frac{1}{2}, -\frac{1}{2} \right\rangle = 0$$

Cálculo de la realización de S_x

$$S = \frac{\hbar}{2}\sigma \qquad S = (S_x, S_y, S_z) \qquad \sigma = (\sigma_x, \sigma_y, \sigma_z)$$

$$S_+ = \begin{pmatrix} 0 & \hbar \\ 0 & 0 \end{pmatrix} \qquad S_+ = \begin{pmatrix} 0 & 0 \\ \hbar & 0 \end{pmatrix}$$

$$S_x = \frac{S_+ + S_-}{2} = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \frac{\hbar}{2}\sigma_x$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Matriz S_z

$$S=\frac{\hbar}{2}\sigma \qquad S=(S_x,S_y,S_z) \qquad \sigma=(\sigma_x,\sigma_y,\sigma_z)$$

$$S_z=\begin{pmatrix} \langle \frac{1}{2},\frac{1}{2}|S_z|\frac{1}{2},\frac{1}{2}\rangle & \langle \frac{1}{2},\frac{1}{2}|S_z|\frac{1}{2},-\frac{1}{2}\rangle \\ \langle \frac{1}{2},-\frac{1}{2}|S_z|\frac{1}{2},\frac{1}{2}\rangle & \langle \frac{1}{2},-\frac{1}{2}|S_z|\frac{1}{2},-\frac{1}{2}\rangle \end{pmatrix}$$

$$S_z=\begin{pmatrix} \frac{\hbar}{2} & 0 \\ 0 & \frac{-\hbar}{2} \end{pmatrix}=\frac{\hbar}{2}\sigma_z \quad \Longrightarrow \quad \sigma_z=\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$S_z||s\,m_s\rangle=\hbar\,m_s||s\,m_s\rangle$$

$$\langle s\,m_s|s'\,m_s'\rangle=\delta_{s\,s'}\,\delta_{m_s\,m_s'}$$

Matriz S_y

$$S=\frac{\hbar}{2}\sigma \qquad S=(S_x,S_y,S_z) \qquad \sigma=(\sigma_x,\sigma_y,\sigma_z)$$

$$S_y=\begin{pmatrix} \langle \frac{1}{2},\frac{1}{2}|S_y|\frac{1}{2},\frac{1}{2}\rangle & \langle \frac{1}{2},\frac{1}{2}|S_y|\frac{1}{2},-\frac{1}{2}\rangle \\ \langle \frac{1}{2},-\frac{1}{2}|S_y|\frac{1}{2},\frac{1}{2}\rangle & \langle \frac{1}{2},-\frac{1}{2}|S_y|\frac{1}{2},-\frac{1}{2}\rangle \end{pmatrix}$$

$$S_+=S_x+iS_y$$
$$S_-=S_x-iS_y$$

$$\Rightarrow \quad S_y=\frac{S_+-S_-}{2i}$$

$$S_\pm|s,m_s\rangle=\hbar\sqrt{s(s+1)-m_s(m_s\pm 1)}\,|s,m_s\pm 1\rangle$$

$$S_y=-i\begin{pmatrix} 0 & \frac{\hbar}{2} \\ \frac{-\hbar}{2} & 0 \end{pmatrix}=\frac{\hbar}{2}\sigma_y \quad \Rightarrow \quad \sigma_y=\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

Sobre Simetría de Rotación y Momento Angular...

Suma de momentos angulares

Suma de dos momentos angulares

Momentos angulares

$$[j_1, j_2] = 0$$

$$\{|j_1 m_1\rangle, |j_2 m_2\rangle\}$$

$$\mathbf{j}_1^2 |j_1 m_1\rangle = j_1(j_1+1)\hbar^2 |j_1 m_1\rangle$$

$$j_{1,z} |j_1 m_1\rangle = m_1 \hbar |j_1 m_1\rangle$$

$$m_1 = -j_1, \cdots, j_1$$

$$\mathbf{j}_2^2 |j_2 m_2\rangle = j_2(j_2+1)\hbar^2 |j_2 m_2\rangle$$

$$j_{2,z} |j_2 m_2\rangle = m_2 \hbar |j_2 m_2\rangle$$

$$m_2 = -j_2, \cdots, j_2$$

Autovalores individuales

Momentos angulares

$$[j_1, j_2] = 0$$

$$\{ |j_1 m_1\rangle, |j_2 m_2\rangle \}$$

$$\mathbf{j}_1^2 |j_1 m_1\rangle = j_1(j_1+1)\hbar^2 |j_1 m_1\rangle$$

$$j_{1,z} |j_1 m_1\rangle = m_1 \hbar |j_1 m_1\rangle$$

$$\mathbf{j}_2^2 |j_2 m_2\rangle = j_2(j_2+1)\hbar^2 |j_2 m_2\rangle$$

$$j_{2,z} |j_2 m_2\rangle = m_2 \hbar |j_2 m_2\rangle$$

Composición (suma)

$$J = j_1 + j_2$$

$$J = (J_x, J_y, J_z)$$

$$J_x = j_{1,x} + j_{2,x}$$

$$J_y = j_{1,y} + j_{2,y}$$

$$J_z = j_{1,z} + j_{2,z}$$

Verificación que la suma es un momento angular

Momentos angulares

$$j_k^2 \| j_k m_k \rangle = j_k(j_k+1)\hbar^2 \| j_k m_k \rangle$$

$$j_{k,z} \| j_k m_k \rangle = m_k \hbar \| j_k m_k \rangle$$

$$k=1,2$$

$$[j_1, j_2]=0 \qquad \{ \| j_1 m_1 \rangle, \| j_2 m_2 \rangle \}$$

Momento angular total

$$J = j_1 + j_2 \qquad J=(J_x, J_y, J_z)$$

$$J_x = j_{1,x} + j_{2,x}$$

$$J_y = j_{1,y} + j_{2,y}$$

$$J_z = j_{1,z} + j_{2,z}$$

$$[j_{1,x}, j_{1,y}] = i\hbar j_{3,z}$$

$$[j_{1,x}, j_{2,y}] = 0$$

Mostrarlo para $[J_x, J_y] = i\hbar J_z$

$$[J_\alpha, J_\beta] = i\hbar \sum_\gamma \epsilon_{\alpha\beta\gamma} J_\gamma$$

$$\alpha, \beta, \gamma = (x, y, z)$$

Recordemos Levi-Civita

$$\epsilon_{123} = \epsilon_{231} = \epsilon_{312} = 1$$

$$\epsilon_{132} = \epsilon_{321} = \epsilon_{213} = -1$$

$$\epsilon_{\alpha\beta\gamma} = 0 \, resto$$

Base desacoplada

Base desacoplada

Momentos angulares

$j_1, j_2 \qquad [j_1, j_2] = 0$

$$J = j_1 + j_2$$

$$\mathbf{j}_k^2 \,|\, j_k m_k \rangle = j_k(j_k + 1)\hbar^2 \,|\, j_k m_k \rangle$$

$$j_{k,z} \,|\, j_k m_k \rangle = m_k \hbar \,|\, j_k m_k \rangle$$

$$m_k = -j_k, \,\cdots, j_k \qquad k = 1{,}2$$

Base $\{\,|\, j_1 m_1 j_2 m_2 \rangle\}$

$$|\, j_1 m_1 j_2 m_2 \rangle = |\, j_1 m_1 \rangle \,|\, j_2 m_2 \rangle$$

Ortogonalidad

$$\langle j_1 m_1 j_2 m_2 \,|\, j_1' m_1' j_2' m_2' \rangle =$$
$$= \delta_{j_1 j_1'} \delta_{m_1 m_1'} \delta_{j_2 j_2'} \delta_{m_2 m_2'}$$

Completitud $I = \sum_{m_1 m_2} |\, j_1 m_1 j_2 m_2 \rangle \langle j_1 m_1 j_2 m_2 \,|$

Base desacoplada

Momentos angulares

$j_1, j_2 \qquad [j_1, j_2] = 0$

$$J = j_1 + j_2$$

$$\mathbf{j}_k^2 |j_k m_k\rangle = j_k(j_k+1)\hbar^2 |j_k m_k\rangle$$

$$j_{k,z} |j_k m_k\rangle = m_k \hbar |j_k m_k\rangle \qquad k = 1,2$$

Base $\{ |j_1 m_1 j_2 m_2\rangle = |j_1 m_1\rangle |j_2 m_2\rangle \}$

Sistema completo de operadores que conmutan:

$$\{ j_1^2, j_2^2, j_{1,z}, j_{2,z} \}$$

Ejemplo: Función de onda de una partícula con spin

$$\phi_{nlm_lm_s}(\bar{r}, s) = R_{nl}(r)\, Y_{lm_l}(\hat{r})\, \chi_{sm_s}$$

$$\mathbf{j}_1 = \mathbf{l} \qquad [\mathbf{l}, \mathbf{s}] = 0$$
$$\mathbf{j}_2 = \mathbf{s}$$

Kets

$$\langle r \,|\, nl \rangle = R_{nl}(r) \longrightarrow |\, nl \rangle$$

$$\langle \hat{r} \,|\, lm_l \rangle = Y_{lm_l}(\hat{r}) \longrightarrow |\, lm_l \rangle$$

$$\langle \sigma \,|\, sm_s \rangle = \chi_{sm_s}(\sigma) \longrightarrow |\, sm_s \rangle$$

Base desacoplada

$$\{\, |\, nlm_l sm_s \rangle = |\, nl \rangle\, |\, lm_l \rangle\, |\, sm_s \rangle \,\}$$

Uso del álgebra angular

Función de onda desacoplada

$$\langle r | nl \rangle = R_{nl}(r)$$

$$\phi_{nlm_lm_s}(\bar{r}, s) = R_{nl}(r)\, Y_{lm_l}(\hat{r})\, \chi_{sm_s}$$

$$\langle \hat{r} | lm_l \rangle = Y_{lm_l}(\hat{r})$$

$$\langle \sigma | sm_s \rangle = \chi_{sm_s}(\sigma)$$

$$\phi_{nlm_lm_s}(\bar{r}, s) = \langle r | nl \rangle \langle \hat{r} | lm_l \rangle \langle \sigma | sm_s \rangle$$

$$\langle \boldsymbol{r}\sigma | nlm_lsm_s \rangle = \langle r | nl \rangle \langle \hat{r} | lm_l \rangle \langle \sigma | sm_s \rangle$$

Base desacoplada

$$\{\, | nlm_lsm_s \rangle = | nl \rangle \, | lm_l \rangle \, | sm_s \rangle \}$$

Base angular y spin

$$\{\, | lm_lsm_s \rangle = | lm_l \rangle \, | sm_s \rangle \}$$

Autovalores

Función de onda desacoplada

<u>Base angular y spin</u>

$$\{ |lm_l s m_s\rangle = |lm_l\rangle |sm_s\rangle \}$$

Momentos angulares orbital $\boldsymbol{l}$ $(\hbar = 1)$

$$\boldsymbol{l}^2 |lm_l\rangle = l(l+1) |lm_l\rangle \quad l = 0, 1, 2, \cdots$$

$$l_z |lm_l\rangle = m_l |lm_l\rangle \qquad m_l = -l, \cdots, l$$

Momentos angulares de spin $\boldsymbol{s}$ $(\hbar = 1)$

$$\boldsymbol{s}^2 |sm_s\rangle = s(s+1) |sm_s\rangle = \frac{3}{4} |sm_s\rangle$$

$$s_z |sm_s\rangle = m_s |sm_s\rangle \qquad m_s = -\frac{1}{2}, \frac{1}{2}$$

Autovalores

Función de onda desacoplada

$$\phi_{nlm_lm_s}(\bar{r}, s) = R_{nl}(r)\, Y_{lm_l}(\hat{r})\, \chi_{sm_s}$$

$$\boldsymbol{l}^2 \phi_{nlm_lm_s}(\bar{r}, s) = l(l+1)\phi_{nlm_lm_s}(\bar{r}, s) \qquad l = 0, 1, 2, \cdots$$

$$l_z \phi_{nlm_lm_s}(\bar{r}, s) = m_l\, \phi_{nlm_lm_s}(\bar{r}, s) \qquad m_l = -\, l, \cdots, l$$

$$\boldsymbol{s}^2 \phi_{nlm_lm_s}(\bar{r}, s) = \frac{3}{4}\phi_{nlm_lm_s}(\bar{r}, s)$$

$$s_z \phi_{nlm_lm_s}(\bar{r}, s) = m_s\, \phi_{nlm_lm_s}(\bar{r}, s) \qquad m_s = -\frac{1}{2}, \frac{1}{2}$$

Esta ecuaciones van a ser útiles al resolver la ecuación de Schroeding

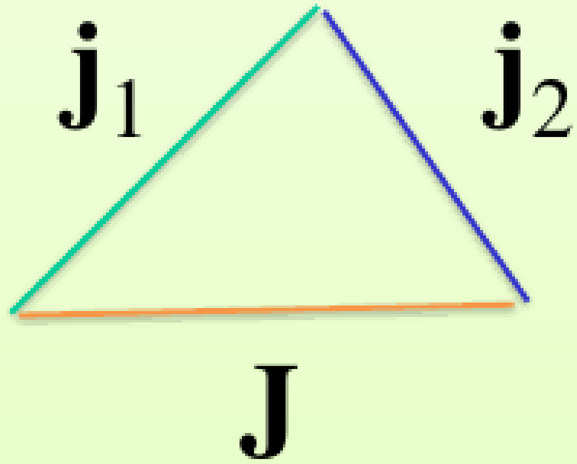
Base acoplada

Base acoplada

Momentos angulares

$\mathbf{j}_1, \mathbf{j}_2 \qquad [\mathbf{j}_1, \mathbf{j}_2] = 0$

$$\mathbf{J} = \mathbf{j}_1 + \mathbf{j}_2$$



Base $\{\, |j_1 j_2 j m\rangle \}$

$$|j_1 - j_2| \leq j \leq j_1 + j_2$$

$$m = -j,\ -j+1,\ \cdots,\ j-1,\ j$$

Ortogonalidad y completitud

$$\langle j_1 j_2 j m \,|\, j_1 j_2 j' m' \rangle = \delta_{jj'}\,\delta_{mm'}$$

$$I = \sum_{jm} |j_1 j_2 j m\rangle\langle j_1 j_2 j m|$$

Base acoplada

Momentos angulares

$\mathbf{j}_1, \mathbf{j}_2 \qquad [\mathbf{j}_1, \mathbf{j}_2] = 0$

$$\mathbf{J} = \mathbf{j}_1 + \mathbf{j}_2$$

Base $\{\,|j_1 j_2 j m\rangle\}$

$\mathbf{j}_1$ $\mathbf{j}_2$

$\mathbf{J}$

Rangos

$$|j_1 - j_2| \leq j \leq j_1 + j_2$$

$$m = -j,\ -j+1,\ \cdots,\ j-1,\ j$$

Autovalores y autovectores

$$\mathbf{j}_1^2\,|j_1 j_2 j m\rangle = j_1(j_1+1)\hbar^2\,|j_1 j_2 j m\rangle$$

$$\mathbf{j}_2^2\,|j_1 j_2 j m\rangle = j_2(j_2+1)\hbar^2\,|j_1 j_2 j m\rangle$$

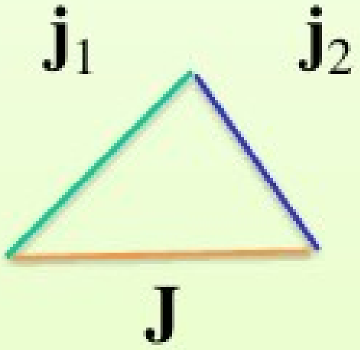
$$\mathbf{J}^2\,|j_1 j_2 j m\rangle = j(j+1)\hbar^2\,|j_1 j_2 j m\rangle$$

$$J_z\,|j_1 j_2 j m\rangle = m\hbar\,|j_1 j_2 j m\rangle$$

Sistema completo de operadores que conmutan:

$$\{\mathbf{j}_1^2, \mathbf{j}_2^2, \mathbf{J}^2, J_z\}$$

Cómo construir la base acoplada



Momentos angulares

$\mathbf{j}_1, \mathbf{j}_2 \qquad [\mathbf{j}_1, \mathbf{j}_2] = 0 \qquad \mathbf{J} = \mathbf{j}_1 + \mathbf{j}_2$

Base desacoplada

$\{\,|j_1 m_1 j_2 m_2\rangle\}$

Completitud

$$I = \sum_{m_1 m_2} |j_1 m_1 j_2 m_2\rangle\langle j_1 m_1 j_2 m_2|$$

Base acoplada

$\{\,|j_1 j_2 j m\rangle\}$

Completitud

$$I = \sum_{jm} |j_1 j_2 j m\rangle\langle j_1 j_2 j m|$$

Construcción: $$I\,|j_1 j_2 j m\rangle = \sum_{m_1 m_2} |j_1 m_1 j_2 m_2\rangle\langle j_1 m_1 j_2 m_2 | j_1 j_2 j m\rangle$$

Construcción de vectores acoplados

Momentos angulares

$$\mathbf{j}_1, \mathbf{j}_2 \quad [\mathbf{j}_1, \mathbf{j}_2] = 0$$

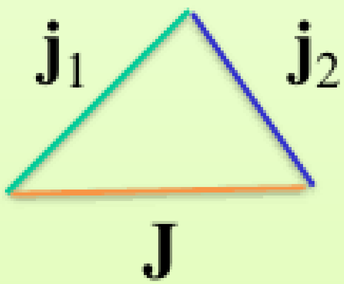
$$\mathbf{J} = \mathbf{j}_1 + \mathbf{j}_2$$

Base desacoplada

$$\{\, |j_1 m_1 j_2 m_2\rangle \}$$

Base acoplada

$$\{\, |j_1 j_2 j m\rangle \}$$



$$|j_1 j_2 j m\rangle = I\, |j_1 j_2 j m\rangle$$

$$|j_1 j_2 j m\rangle = \sum_{m_1 m_2} \langle j_1 m_1 j_2 m_2 | j_1 j_2 j m\rangle\, |j_1 m_1 j_2 m_2\rangle$$

Simplificación en la notación

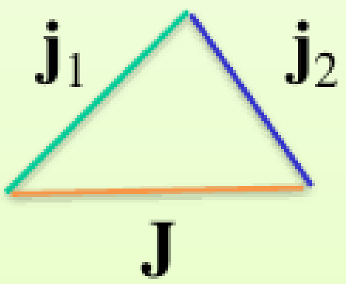
$$|j_1 j_2 j m\rangle = \sum_{m_1 m_2} \langle j_1 m_1 j_2 m_2 | j m\rangle\, |j_1 m_1 j_2 m_2\rangle$$

Coeficientes de Clebsch-Gordan

$$\langle j_1 m_1 j_2 m_2 | j m\rangle \equiv \langle j_1 m_1 j_2 m_2 | j_1 j_2 j m\rangle$$

Propiedades de los coeficientes de Clebsch-Gordan

Propiedades de los Clebsch-Gordan

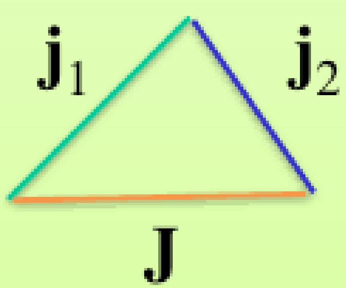


$$m = -j,\ -j+1,\ \cdots,\ j-1, j$$

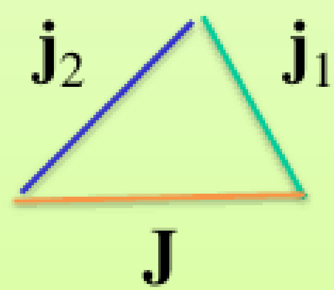
$$|j_1 - j_2| \le j \le j_1 + j_2$$

$$\langle j_1 m_1 j_2 m_2 | jm \rangle = 0 \qquad m_1 + m_2 \neq m$$

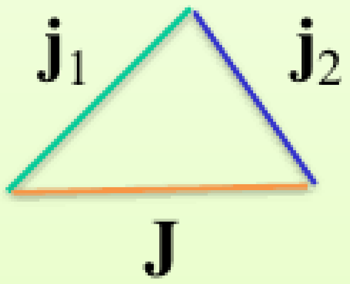
$$\langle j_1 m_1 j_2 m_2 | jm \rangle = (-)^{j_1+j_2-j} \langle j_2 m_2 j_1 m_1 | jm \rangle$$



$$= (-)^{j_1+j_2-j}$$



Propiedades de los Clebsch-Gordan



$$m = -j,\ -j+1,\ \cdots,\ j-1,\ j$$

$$|j_1 - j_2| \le j \le j_1 + j_2$$

$$\hat{j} = \sqrt{2j+1}$$

$$\langle j_1 m_1 j_2 m_2 | jm \rangle = (-)^{j_1+j_2-j} \langle j_1 - m_1 j_2 - m_2 | j - m \rangle$$

$$\langle j_1 m_1 j_2 m_2 | jm \rangle = (-)^{j_1-m_1} \frac{\hat{j}}{\hat{j}_1} \langle j_1 m_1 j - m | j_2 - m_2 \rangle$$

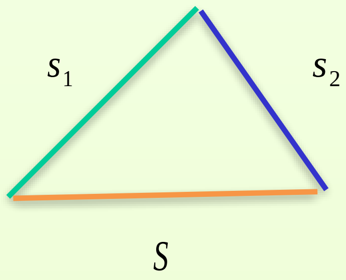
$$\langle jmj - m | 00 \rangle = \frac{(-)^{j-m}}{\hat{j}}$$

$$\langle jm00 | jm \rangle = 1$$

Convención de fase de Condon-Shortley

Acople de dos momentos angulares: spin 1/2

Función de onda de dos fermiones acoplados



$$\mathbf{s}_k^2 \,|\, s_k m_{s_k}\rangle = \frac{3}{4}\,|\, s_k m_{s_k}\rangle \qquad k = 1,2$$

$$\mathbf{s}_{k,z} \,|\, s_k m_{s_k}\rangle = m_k \,|\, s_k m_{s_k}\rangle$$

$$0 \le S \le 1$$

$$S = 0,1$$

$$M_S = -S, 0, S$$

Representación abstracta

$$|\, S M_S\rangle = \sum_{m_{s_1}, m_{s_2}} \langle s_1 m_{s_1} s_2 m_{s_2} \,|\, S M_S\rangle \,|\, s_1 m_{s_1}\rangle \,|\, s_2 m_{s_2}\rangle$$

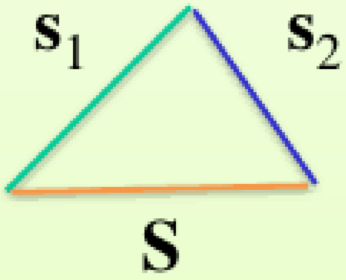
Representación espinor

$$\langle \sigma \,|\, s m_s\rangle = \chi_{s m_s}(\sigma)$$

$$\langle \sigma_1 \sigma_2 \,|\, S M_S\rangle = \sum_{m_{s_1}, m_{s_2}} \langle s_1 m_{s_1} s_2 m_{s_2} \,|\, S M_S\rangle \langle \sigma_1 \,|\, s_1 m_{s_1}\rangle \langle \sigma_2 \,|\, s_2 m_{s_2}\rangle$$

$$\chi_{S M_S}(\sigma_1, \sigma_2) = \sum_{m_{s_1}, m_{s_2}} \langle s_1 m_{s_1} s_2 m_{s_2} \,|\, S M_S\rangle \chi_{s_1 m_{s_1}}(\sigma_1) \chi_{s_2 m_{s_2}}(\sigma_2)$$

Singlete y triplete



$$S = 0{,}1$$

$$M_{S=0} = 0$$

$$M_{S=1} = -1{,}0{,}1$$

$$\hat{s}_1 = \hat{s}_2 = \sqrt{2}$$

$$\langle 1/2\,1/2,\,1/2\ -1/2\,|\,00\rangle = \frac{1}{\sqrt{2}}$$

$$\langle j_1 m_1 j_2 m_2\,|\,jm\rangle = (-)^{j_1+j_2-j}\langle j_1 -m_1 j_2 -m_2\,|\,j-m\rangle$$

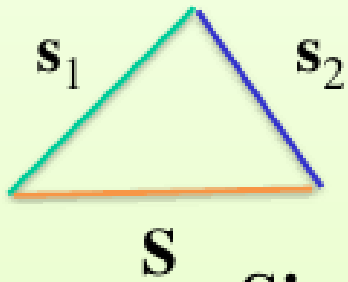
$$\langle 1/2\ -1/2,\,1/2\,1/2\,|\,00\rangle = -\frac{1}{\sqrt{2}}$$

$$\langle 1/2\ -1/2,\,1/2\,1/2\,|\,00\rangle = (-)^{1/2+1/2-0}\langle 1/2\,1/2,\,1/2\ -1/2\,|\,00\rangle$$

$$\langle 1/2\,1/2,\,1/2\ -1/2\,|\,10\rangle = \langle 1/2\ -1/2,\,1/2\,1/2\,|\,10\rangle = \frac{1}{\sqrt{2}}$$

$$\langle 1/2\,1/2,\,1/2\,1/2\,|\,10\rangle = \langle 1/2\ -1/2,\,1/2\ -1/2\,|\,10\rangle = 1$$

Función de onda de dos fermiones acoplados



Singlete

$$\chi_{0,0}(\sigma_1,\sigma_2) = \sum_{m_{s_1},m_{s_2}} \langle s_1 m_{s_1} s_2 m_{s_2} | 0{,}0 \rangle \chi_{s_1 m_{s_1}}(\sigma_1)\chi_{s_2 m_{s_2}}(\sigma_2)$$

$$\langle s_1 m_{s_1} s_2 m_{s_2} | 0{,}0 \rangle = \delta_{s_1,s_2}\delta_{m_1,-m_2}\frac{(-)^{s_1-m_1}}{\hat{s}_1}$$

$$S = 0 \Rightarrow M_S = 0$$

Impar

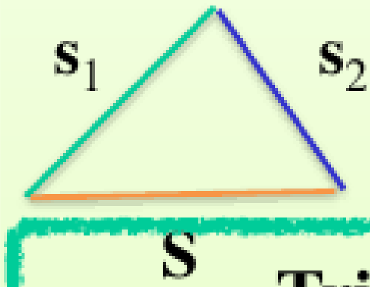
$$\sigma_1 \leftrightarrows \sigma_2$$

$$\hat{s} = \sqrt{2s+1} = \sqrt{2}$$

$$\chi_{0,0}(\sigma_1,\sigma_2) = \frac{1}{\sqrt{2}}\left(\chi_{s,1/2}(\sigma_1)\chi_{s,-1/2}(\sigma_2) - \chi_{s,1/2}(\sigma_2)\chi_{s,-1/2}(\sigma_1)\right)$$

$$\chi_{0,0}(\sigma_2,\sigma_1) = \frac{1}{\sqrt{2}}\left(\chi_{s,1/2}(\sigma_2)\chi_{s,-1/2}(\sigma_1) - \chi_{s,1/2}(\sigma_1)\chi_{s,-1/2}(\sigma_2)\right) = -\chi_{0,0}(\sigma_1,\sigma_2)$$

Función de onda de dos fermiones acoplados



$$\chi_{1,M_S}(\sigma_1,\sigma_2) = \sum_{m_{s_1},m_{s_2}} \langle s_1 m_{s_1} s_2 m_{s_2} \,|\, 1, M_S = m_1 + m_2 \rangle \chi_{s_1 m_{s_1}}(\sigma_1)\chi_{s_2 m_{s_2}}(\sigma_2)$$

Triplete

$$S = 1 \Rightarrow M_S = -1, 0, 1$$

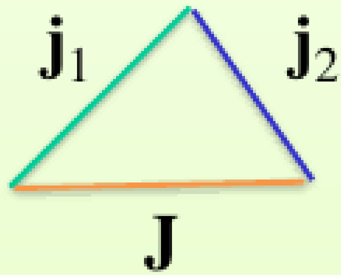
Par $\sigma_1 \leftrightarrows \sigma_2$

$$\chi_{1,0}(\sigma_1,\sigma_2) = \sum_{m_{s_1},m_{s_2}} \langle s_1 m_{s_1} s_2 m_{s_2} \,|\, 1, 0 \rangle \chi_{s_1 m_{s_1}}(\sigma_1)\chi_{s_2 m_{s_2}}(\sigma_2)$$

$$\chi_{1,0}(\sigma_1,\sigma_2) = \frac{1}{\sqrt{2}}\left(\chi_{s,1/2}(\sigma_1)\chi_{s,-1/2}(\sigma_2) + \chi_{s,1/2}(\sigma_2)\chi_{s,-1/2}(\sigma_1)\right)$$

$$\chi_{1,0}(\sigma_2,\sigma_1) = \frac{1}{\sqrt{2}}\left(\chi_{s,1/2}(\sigma_2)\chi_{s,-1/2}(\sigma_1) + \chi_{s,1/2}(\sigma_1)\chi_{s,-1/2}(\sigma_2)\right) = \chi_{0,0}(\sigma_1,\sigma_2)$$

Propiedades de los Clebsch-Gordan



$$|j_1 - j_2| \leq j \leq j_1 + j_2$$

$$m = -j,\ -j+1,\ \cdots,\ j-1,\ j$$

$$\sum_{m_1 m_2} \langle j_1 m_1 j_2 m_2 | jm \rangle \langle j_1 m_1 j_2 m_2 | j'm' \rangle = \delta_{jj'} \delta_{mm'}$$

$$\sum_{m_1 m_2} \langle j_1 m_1 j_2 m_2 | jm \rangle \langle j_1 m_1 j_2 m_2 | m \rangle = 1$$

$$|jm\rangle = \sum_{m_1 m_2} \langle j_1 m_1 j_2 m_2 | jm \rangle \, | j_1 m_1 j_2 m_2 \rangle \qquad \langle jm | = \sum_{m_1 m_2} \langle j_1 m_1 j_2 m_2 | jm \rangle \langle j_1 m_1 j_2 m_2 |$$

$$\delta_{jj'} \delta_{mm'} = \langle jm | j'm' \rangle = \sum_{m_1 m_2} \sum_{m_1' m_2'} \langle j_1 m_1 j_2 m_2 | jm \rangle \langle j_1 m_1' j_2 m_2' | jm \rangle \langle j_1 m_1 j_2 m_2 | j_1 m_1' j_2 m_2' \rangle$$

$$= \sum_{m_1 m_2} \langle j_1 m_1 j_2 m_2 | jm \rangle \langle j_1 m_1 j_2 m_2 | j'm' \rangle$$

Aplicación a la función de onda de una partícula

Uso del álgebra angular

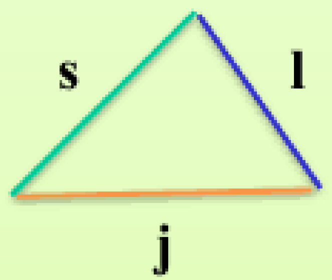
Función de onda acoplada

$$\psi_{nljm}(\boldsymbol{r}, s) = R_{nlj}(r)[\chi_s Y_l(\theta, \phi)]_{jm}$$

$$[\chi_s Y_l(\theta, \phi)]_{jm} = \sum_{m_s, m_l} \langle s m_s l m_l | j m \rangle \chi_{s m_s} Y_{l m_l}(\theta, \phi)$$

$$[\chi_s Y_l(\theta, \phi)]_{jm} = \mathscr{Y}_{ljm}(\hat{r})$$

Acople s-l



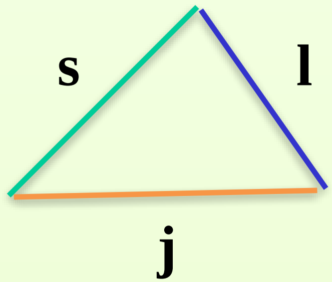
$$|l - s| \leq j \leq l + s \longrightarrow \quad j = l \pm \frac{1}{2}$$

$$m = -j, -j+1, \cdots, j-1, j$$

Uso del álgebra angular

Función de onda acoplada

$$\psi_{nljm}(\boldsymbol{r}, s) = R_{nlj}(r)[\chi_s Y_l(\theta, \phi)]_{jm}$$



Kets

$$\psi_{nljm}(\boldsymbol{r}) = \langle \boldsymbol{r} \,|\, nljm \rangle \longrightarrow |\, nljm \rangle$$

$$\langle r \,|\, nlj \rangle = R_{nl}(r) \to |\, nlj \rangle$$

$$\langle \hat{r} \,|\, sl, jm \rangle = \mathscr{Y}_{ljm}(\hat{r}) \longrightarrow |\, sl, jm \rangle \qquad |\, ljm \rangle \equiv |\, sl, jm \rangle$$

Base acoplada

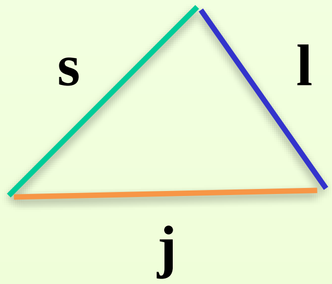
$$\{ \,|\, nljm \rangle = |\, nlj \rangle \,|\, ljm \rangle \}$$

Uso del álgebra angular

Función de onda acoplada

$$\psi_{nljm}(\boldsymbol{r}, s) = R_{nlj}(r)[\chi_s Y_l(\theta, \phi)]_{jm}$$

$$\psi_{nljm}(\boldsymbol{r}) = \langle \boldsymbol{r} \,|\, nljm \rangle$$



Autovectores

$$\boldsymbol{l}^2 \,|\, nljm \rangle = l(l+1) \,|\, nljm \rangle$$

$$\boldsymbol{s}^2 \,|\, nljm \rangle = s(s+1) \,|\, nljm \rangle = \frac{3}{4} \,|\, nljm \rangle$$

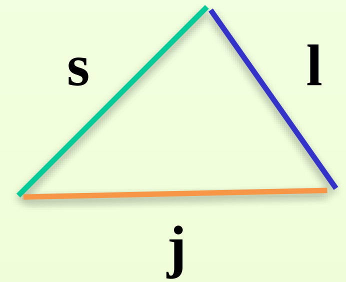
$$\boldsymbol{j}^2 \,|\, nljm \rangle = j(j+1) \,|\, nljm \rangle$$

$$j_z \,|\, nljm \rangle = m \,|\, nljm \rangle$$

Uso del álgebra angular

Función de onda acoplada

$$\psi_{nljm}(\boldsymbol{r}, s) = R_{nlj}(r)[\chi_s Y_l(\theta, \phi)]_{jm}$$



Autovectores

$$\boldsymbol{l}^2 \psi_{nljm}(\boldsymbol{r}, s) = l(l+1)\, \psi_{nljm}(\boldsymbol{r}, s)$$

$$\boldsymbol{s}^2 \psi_{nljm}(\boldsymbol{r}, s) = \frac{3}{4}\, \psi_{nljm}(\boldsymbol{r}, s)$$

$$\boldsymbol{j}^2 \psi_{nljm}(\boldsymbol{r}, s) = j(j+1)\, \psi_{nljm}(\boldsymbol{r}, s)$$

$$j_z \psi_{nljm}(\boldsymbol{r}, s) = m\, \psi_{nljm}(\boldsymbol{r}, s)$$

Volveremos a esto al resolver la ecuación de Schroedinger

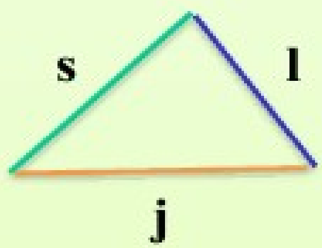
Sobre el orden del acople

Cambio de orden del acople

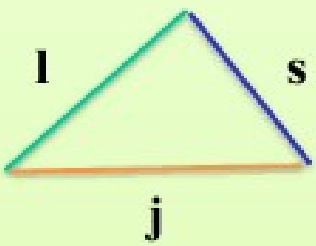
Función de onda acoplada

$$\psi_{nsljm}(\boldsymbol{r}, s) = R_{nlj}(r)[\chi_s Y_l(\theta, \phi)]_{jm}$$

Orden *sl*



Orden *ls*



$$\psi_{nlsjm}(\boldsymbol{r}, s) = R_{nlj}(r)[Y_l(\theta, \phi)\chi_s]_{jm}$$

$$= (\,-\,)^{s+l-j} R_{nlj}(r)[\chi_s Y_l(\theta, \phi)]_{jm}$$

$$\psi_{nlsjm}(\boldsymbol{r}, s) = (\,-\,)^{s+l-j}\, \psi_{nsljm}(\boldsymbol{r}, s)$$

Aplicación: s=1/2, l=1, j=3/2, 1/2

Autovalores del operador

$$l \cdot s$$

Uso del álgebra angular

Autovalores del producto escalar $\boldsymbol{l} \cdot \boldsymbol{s}$

Base acoplada

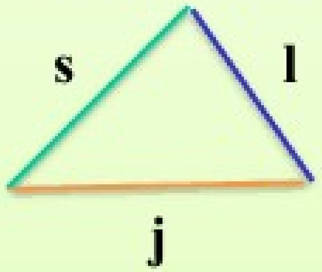
$$\boldsymbol{l}^2 |nljm\rangle = l(l+1)|nljm\rangle$$

$$\boldsymbol{s}^2 |nljm\rangle = s(s+1)|nljm\rangle$$

$$\boldsymbol{j}^2 |nljm\rangle = j(j+1)|nljm\rangle$$

$$j_z |nljm\rangle = m|nljm\rangle$$

Acople s-l



$$\boldsymbol{j}^2 = \boldsymbol{s}^2 + \boldsymbol{l}^2 + 2\boldsymbol{l} \cdot \boldsymbol{s}$$

$$\boldsymbol{l} \cdot \boldsymbol{s} = \frac{\boldsymbol{j}^2 - \boldsymbol{s}^2 - \boldsymbol{l}^2}{2}$$

$[\bar{l}, \bar{s}] = 0$

$$\boldsymbol{l} \cdot \boldsymbol{s}\,|nljm\rangle = \frac{j(j+1) - l(l+1) - 3/4}{2}|nljm\rangle$$

Resumen:
Comparación función de onda no acoplada y acoplada

Uso del álgebra angular

Funciones de onda desacoplada y acoplada

Función de onda desacoplada

$$\phi_{nlm_lm_s}(\bar{r}, s) = R_{nl}(r)\, Y_{lm_l}(\hat{r})\, \chi_{sm_s}$$

$$\phi_{nlm_lm_s}(\bar{r}, s) = \langle \boldsymbol{r} \,|\, nlm_lsm_s \rangle$$

Función de onda acoplada

$$\psi_{nljm}(\boldsymbol{r}, s) = R_{nlj}(r)[\chi_s Y_l(\theta, \phi)]_{jm}$$

$$\psi_{nljm}(\boldsymbol{r}) = \langle \boldsymbol{r} \,|\, nljm \rangle$$

Sistema completo de observables que conmutan

Base desacoplada

$$\{l^2, s^2, l_z, m_z\}$$

Base acoplada

$$\{l^2, s^2, j^2, j_z\}$$

Isospin

Isoespín: formalismo

Función de onda protón $\phi_{proton} = \phi_\alpha(\boldsymbol{x})\begin{pmatrix}1\\0\end{pmatrix} = \phi_\alpha(\boldsymbol{r},s)\zeta_{-1/2}$

Función de onda neutrón $\phi_{neutron} = \phi_\alpha(\boldsymbol{x})\begin{pmatrix}0\\1\end{pmatrix} = \phi_\alpha(\boldsymbol{r},s)\zeta_{1/2}$

Isoespín 1/2 $t=(t_x, t_y, t_z)$

Álgebra momento angular

$$[t_x, t_y] = i t_z$$

Representación: matrices de Pauli $t_k = \frac{1}{2}\tau_k$

$$\tau_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Isoespín: formalismo

Función de onda protón $\phi_{proton} = \phi_\alpha(\boldsymbol{x})\begin{pmatrix}1\\0\end{pmatrix} = \phi_\alpha(\boldsymbol{r}, s)\zeta_{-1/2}$

Función de onda neutrón $\phi_{neutron} = \phi_\alpha(\boldsymbol{x})\begin{pmatrix}0\\1\end{pmatrix} = \phi_\alpha(\boldsymbol{r}, s)\zeta_{1/2}$

Isoespín $t=(t_x, t_y, t_z)$ $[t_x, t_y]=i t_z$

Autovalores

$$t^2\zeta_\mu = \frac{3}{4}\zeta_\mu$$

$$t_z\zeta_{1/2} = \frac{1}{2}\zeta_{1/2}$$

$$t_z\zeta_{-1/2} = -\frac{1}{2}\zeta_{-1/2}$$

neutrones $\zeta_{1/2}$

protones $\zeta_{-1/2}$

Isoespín: transformación n/p

Función de onda protón

$$\zeta_{-1/2} = |1/2, -1/2\rangle$$

$$t_z \zeta_{-1/2} = -\frac{1}{2}\zeta_{-1/2}$$

Función de onda neutrón

$$\zeta_{1/2} = |1/2, 1/2\rangle$$

$$t_z \zeta_{1/2} = \frac{1}{2}\zeta_{1/2}$$

Operadores de crecimiento $t_{\pm} = t_x \pm i\, t_y$

$$t_{\pm}|t t_z\rangle = \sqrt{t(t+1) - t_z(t_z \pm 1)}\,|t, \pm t_z\rangle = \sqrt{\frac{3}{4} - t_z(t_z \pm 1)}\,|t, \pm t_z\rangle$$

$$t_+\zeta_{-1/2} = \zeta_{1/2} \qquad t_+\zeta_{1/2} = 0 \qquad t_-\zeta_{1/2} = \zeta_{-1/2} \qquad t_-\zeta_{-1/2} = 0$$

Decaimiento β

Neutrones $\zeta_{1/2}$

Operador para decaimiento β^-

$$t_-\zeta_{1/2} = \zeta_{-1/2}$$

$$n \rightarrow p + e + \bar{\nu}_e$$

Protones $\zeta_{1/2}$

Operador para decaimiento β^+

$$t_+\zeta_{-1/2} = \zeta_{1/2}$$

$$p \rightarrow n + e + \nu_e$$

Isoespín de dos nucleones

Isoespín de dos nucleones

Isoespín de dos nucleones

$$\zeta_{T,T_z}(1,2) = \sum_{\substack{\mu_1\mu_2 \\ T_z=\mu_1+\mu_2}} \langle 1/2\mu_1 1/2\mu_2 | TT_z \rangle \zeta_{\mu_1}(1)\zeta_{\mu_2}(2)$$

$$T = t(1) + t(2) \qquad T = 0, 1$$

$$T_z = t_z(1) + t_z(2)$$

$$T_Z = -T, \cdots, T$$

Autovalores

$$T^2 \zeta_{TT_z} = T(T+1)\zeta_{TT_z}$$

$$T_z \zeta_{TT_z} = T_z \zeta_{TT_z}$$

Isoespín de dos nucleones

Isoespín de dos nucleones

$$\zeta_{T,T_z}(1,2) = \sum_{\substack{\mu_1\mu_2 \\ T_z=\mu_1+\mu_2}} \langle 1/2\mu_1 1/2\mu_2 | T T_z\rangle \zeta_{\mu_1}(1)\zeta_{\mu_2}(2)$$

❑$^{28}_{14}Si_{14}+dos\ nucleones$

$t_z(1)=1/2\ t_z(2)=1/2$

❑$^{28}_{14}Si_{14}+2n=$❑$^{30}_{14}Si_{16}$

❑$^{28}_{14}Si_{14}+n+p=$❑$^{30}_{15}P_{15}$

$t_z(1)=1/2$

$t_z(2)=-1/2$

❑$^{28}_{14}S_{14}+2p=$❑$^{30}_{16}S_{14}$

$t_z(1)=-1/2\ t_z(2)=-1/2$

Simetría

$[H,T_z]=0$

Simetría aproximada

$[H,T^2]\approx 0$

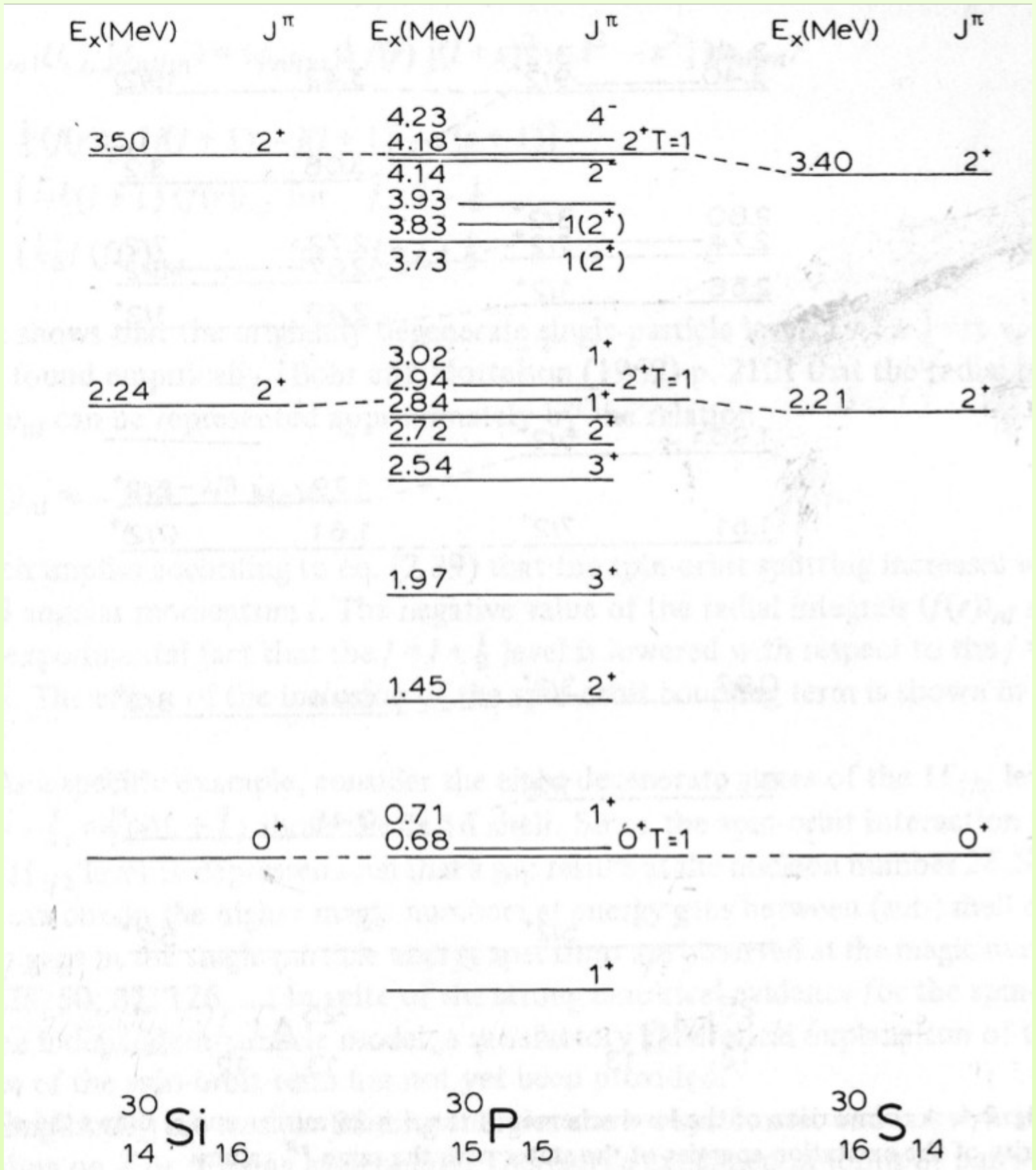
Isoespín de dos nucleones

$${}^{28}_{14}Si_{14} + dos\ nucleones$$

$${}^{28}_{14}Si_{14} + 2n = {}^{30}_{14}Si_{16}$$

$${}^{28}_{14}Si_{14} + n + p = {}^{30}_{15}P_{15}$$

$${}^{28}_{14}S_{14} + 2p = {}^{30}_{16}S_{14}$$

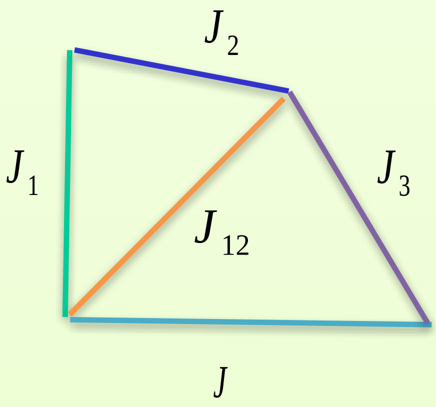


$$[H, T^2] \approx 0 \qquad [H, T_z] = 0$$

Crédito: P. J. Brussard and P. W. M. Glaudemans. Shell-Model Appl. in Nuclear Spectroscopic. 1977

Acople de tres momentos angulares

Acople de tres momentos angulares



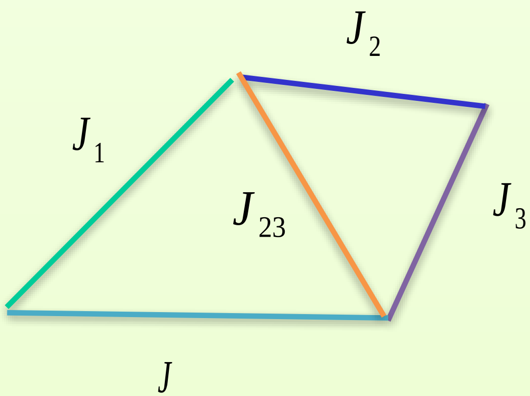
Opciones de acoples

- $\boldsymbol{J}_{12} = \boldsymbol{J}_1 + \boldsymbol{J}_2,\ \boldsymbol{J} = \boldsymbol{J}_{12} + \boldsymbol{J}_3$
- $\boldsymbol{J}_{23} = \boldsymbol{J}_2 + \boldsymbol{J}_3,\ \boldsymbol{J} = \boldsymbol{J}_{23} + \boldsymbol{J}_1$
- $\boldsymbol{J}_{13} = \boldsymbol{J}_1 + \boldsymbol{J}_3,\ \boldsymbol{J} = \boldsymbol{J}_{13} + \boldsymbol{J}_2$

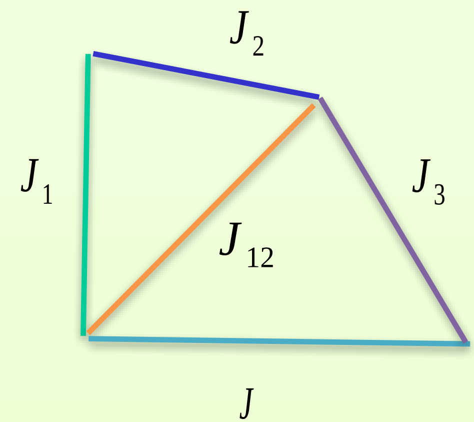
Bases

$$
\begin{aligned}
I &= \sum_{J_{12}} |j_1 j_2 (j_{12}) j_3; jm\rangle\langle j_1 j_2 (j_{12}) j_3; jm| \\
I &= \sum_{J_{23}} |j_1 j_2 j_3 (j_{23}); jm\rangle\langle j_1 j_2 j_3 (j_{23}); jm| \\
I &= \sum_{J_{13}} |j_1 j_3 (j_{13}) j_2; jm\rangle\langle j_1 j_3 (j_{13}) j_2; jm|
\end{aligned}
$$

Cambio de base



$$\sum_{J_{12}} \langle j_1 j_2 (j_{12}) j_3 ; jm \vee j_1 j_2 j_3 (j_{23}) ; jm \rangle$$



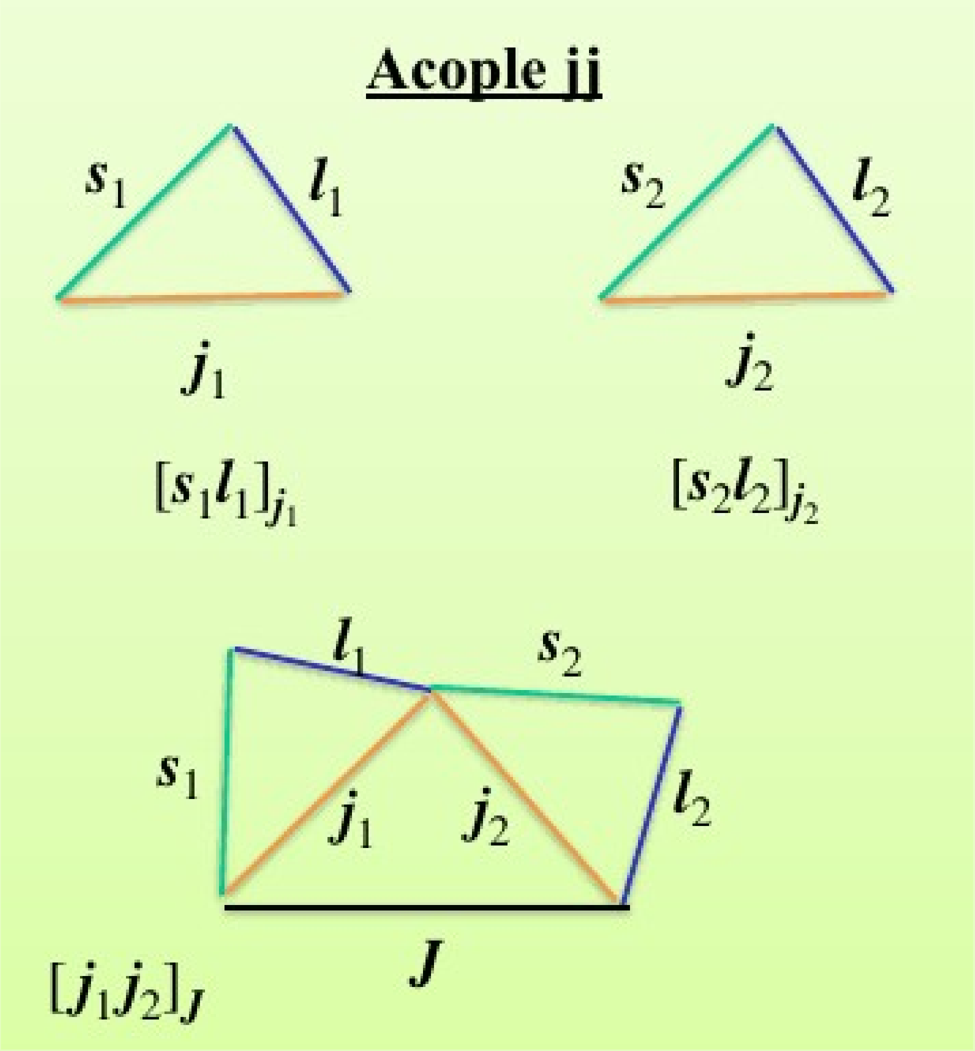
$$\begin{aligned} |j_1 j_2 j_3 (j_{23}); jm\rangle &= \sum_{J_{12}} |j_1 j_2 (j_{12}) j_3; jm\rangle \langle j_1 j_2 (j_{12}) j_3; jm | j_1 j_2 j_3 (j_{23}); jm\rangle \\ &= \sum_{J_{12}} (-)^{j_1+j_2+j_3+j} \hat{j}_{12} \hat{j}_{23} \left\{ \begin{array}{ccc} j_1 & j_2 & j_{12} \\ j_3 & j & j_{23} \end{array} \right\} |j_1 j_2 (j_{12}) j_3; jm\rangle \end{aligned}$$

Coeficiente de Wigner 6j

Acople de cuatro momentos angulares

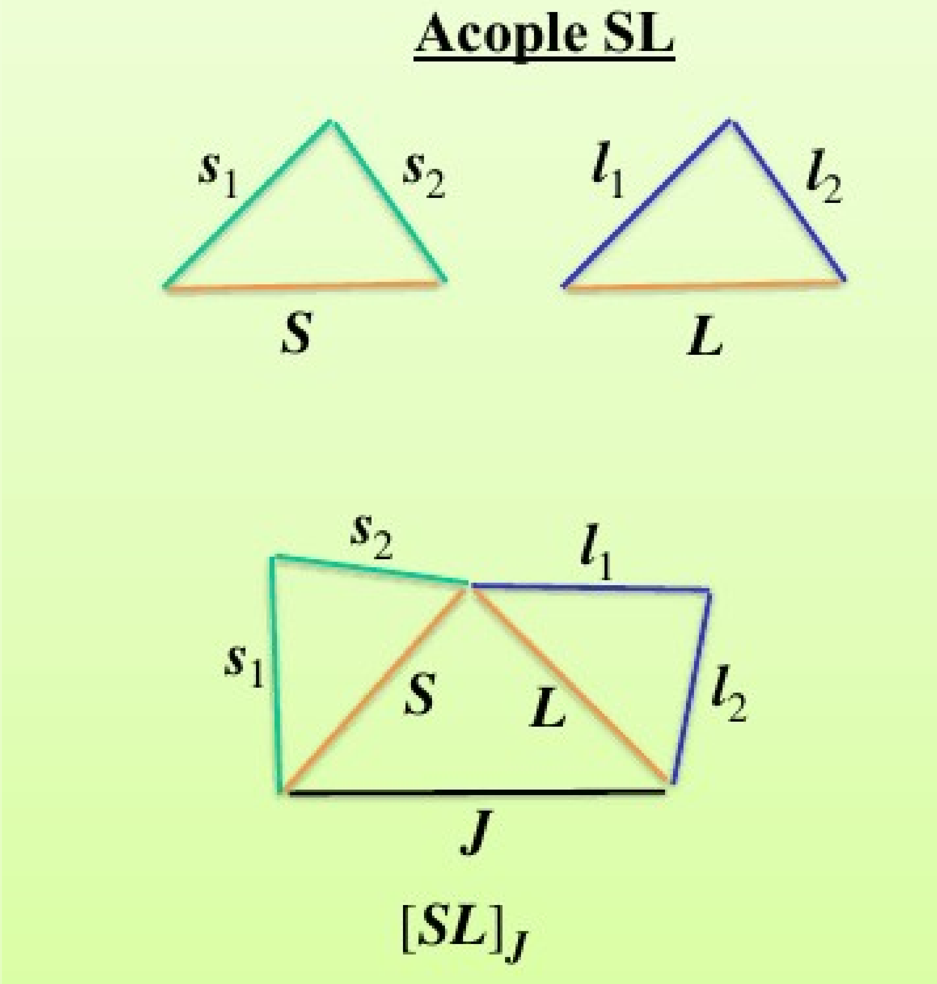
Acople de cuatro momentos angulares

Partícula 1: (s_1, l_1)
Partícula 2: (s_2, l_2)



Acople de cuatro momentos angulares

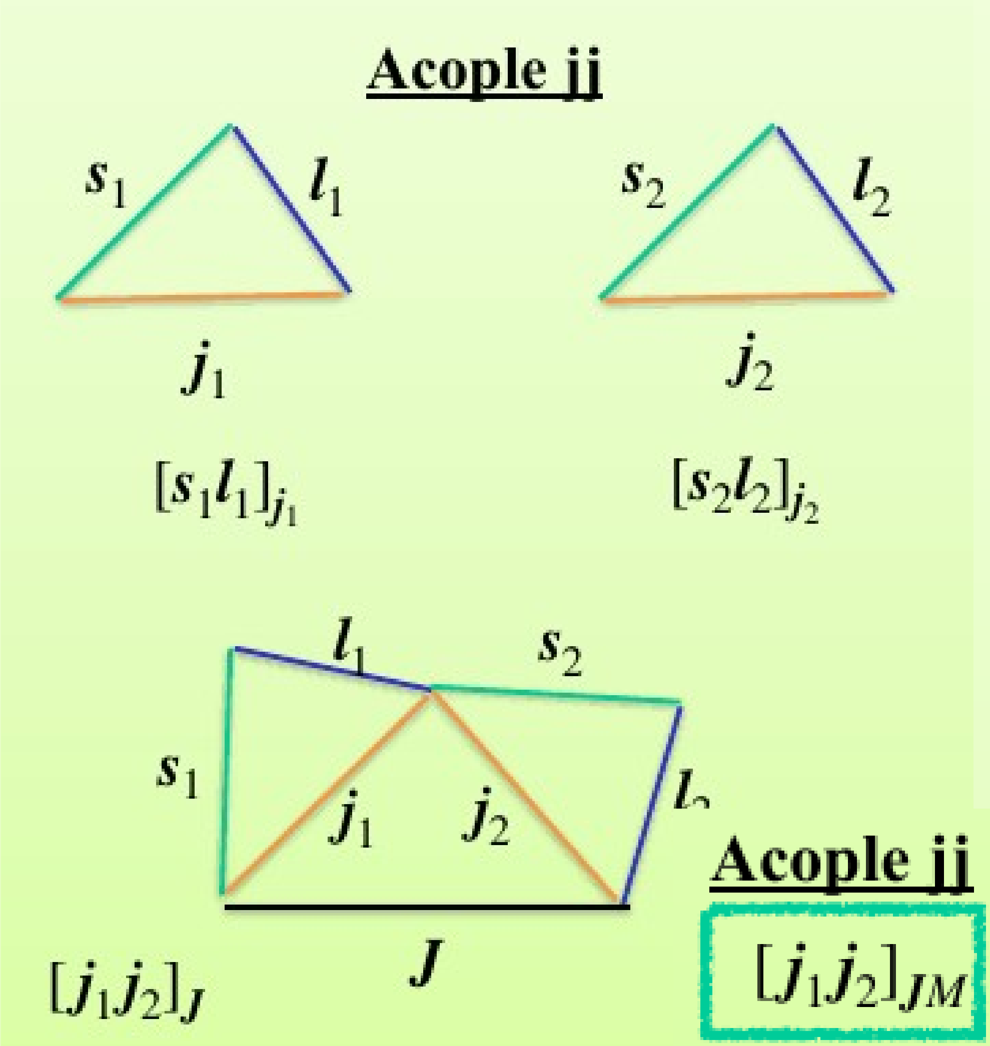
Partícula 1: (s_1, l_1)
Partícula 2: (s_2, l_2)

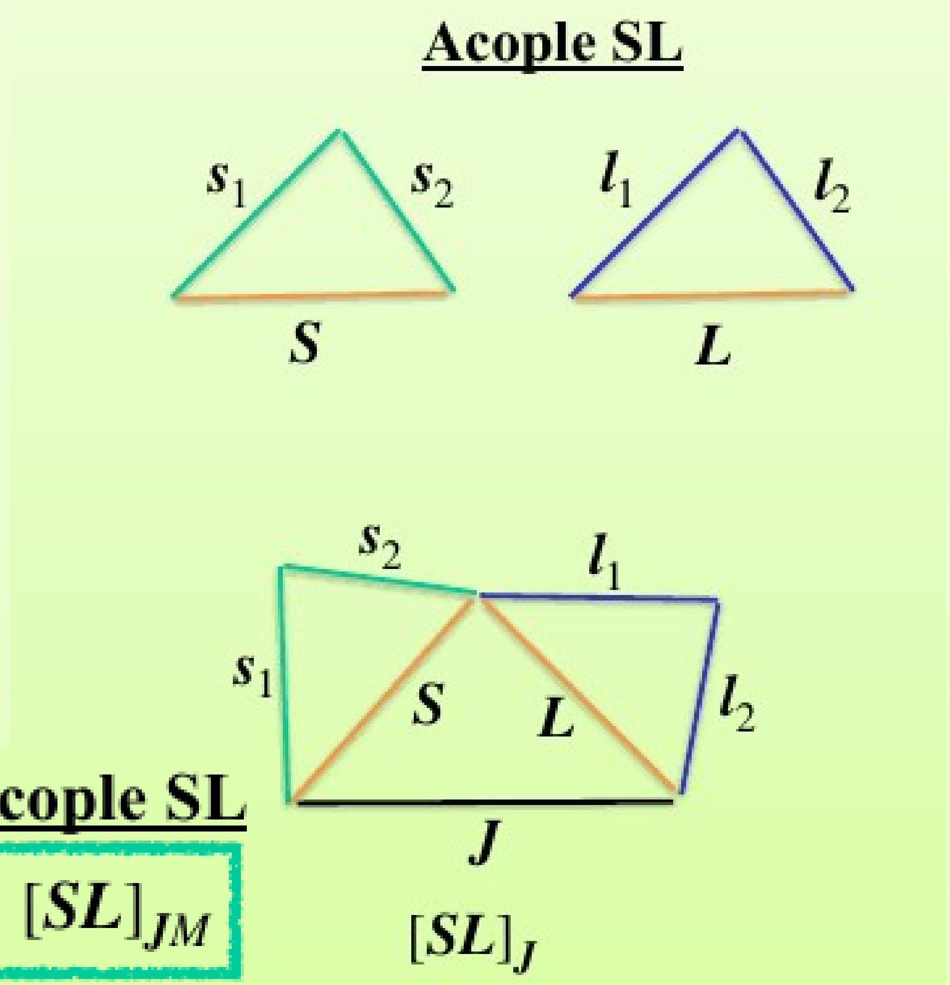


Acople de cuatro momentos angulares

Partícula 1: (s_1, l_1)

Partícula 2: (s_2, l_2)





Acople jj

$[j_1 j_2]_{JM}$

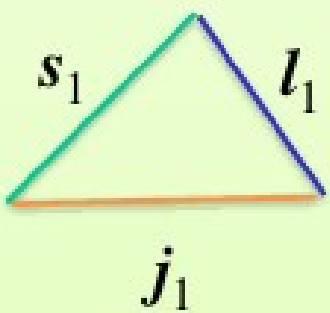
Acople SL

$[SL]_{JM}$

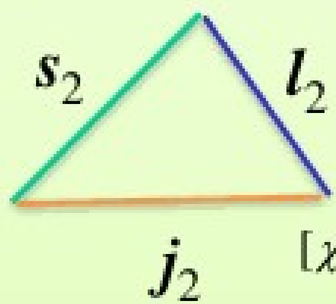
Acople jj con funciones de onda

Partícula 1: (s_1, l_1) **Partícula 2:** (s_2, l_2)

Acople jj



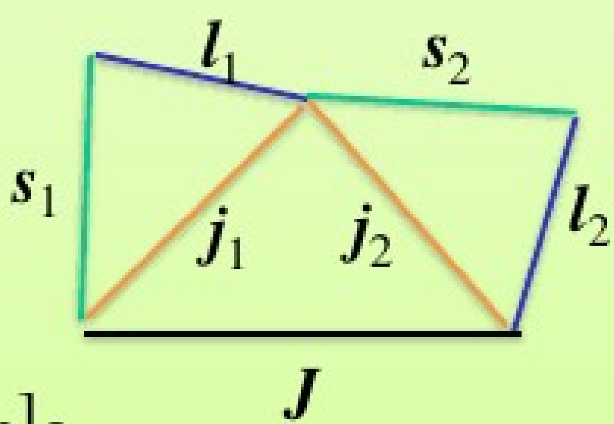
$[s_1 l_1]_{j_1}$



$[s_2 l_2]_{j_2}$

$$[\chi_{s_1} Y_{l_1}(\theta_1, \phi_1)]_{j_1 m_1} = \sum_{m_{s_1}, m_{l_1}} \langle s_1 m_{s_1} l_1 m_{l_1} | j_1 m_1 \rangle \chi_{s_1 m_{s_1}} Y_{l_1 m_{l_1}}(\theta_1, \phi_1)$$

$$[\chi_{s_2} Y_{l_2}(\theta_2, \phi_2)]_{j_2 m_2} = \sum_{m_{s_2}, m_{l_2}} \langle s_2 m_{s_2} l_2 m_{l_2} | j_2 m_2 \rangle \chi_{s_2 m_{s_2}} Y_{l_2 m_{l_2}}(\theta_2, \phi_2)$$



$[j_1 j_2]_J$

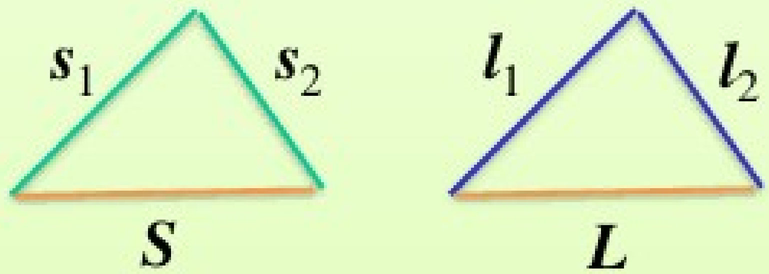
$$\langle \hat{r}_1 \hat{r}_2 | j_1 j_2, jm \rangle = \sum_{m_1, m_2} \langle j_1 m_1 j_2 m_2 | jm \rangle [\chi_{s_1} Y_{l_1}]_{j_1 m_1} [\chi_{s_2} Y_{l_2}]_{j_2 m_2}$$

Acople SL con funciones de onda

Partícula 1: (s_1, l_1) **Partícula 2:** (s_2, l_2)

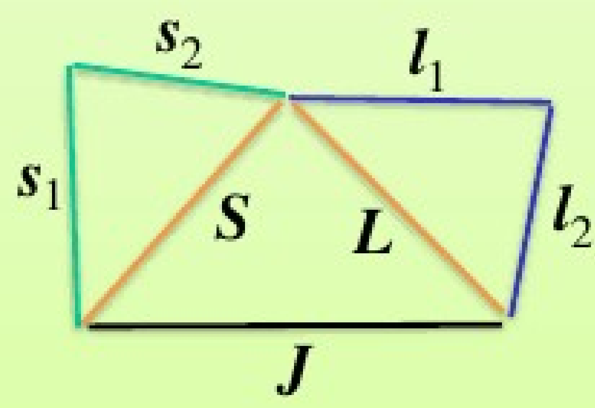
Acople SL

$$\chi_{SM_S}(\sigma_1,\sigma_2) = \sum_{m_{s_1},m_{s_2}} \langle s_1 m_{s_1} s_2 m_{s_2} | SM_S \rangle \chi_{s_1 m_{s_1}}(\sigma_1)\chi_{s_2 m_{s_2}}(\sigma_2)$$



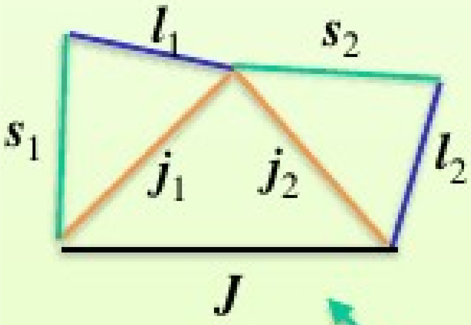
$$\mathbf{Y}_{LM_L}(\hat{r}_1,\hat{r}_2) = \sum_{m_{l_1},m_{l_2}} \langle l_1 m_{l_1} l_2 m_{l_2} | LM_L \rangle Y_{l_1 m_{l_1}}(\hat{r}_1) Y_{l_2 m_{l_2}}(\hat{r}_2)$$

$$\langle \hat{r}_1 \hat{r}_2 | SL, jm \rangle = \sum_{M_S,M_L} \langle SM_S LM_L | jm \rangle \chi_{SM_S} \mathbf{Y}_{LM_L}$$

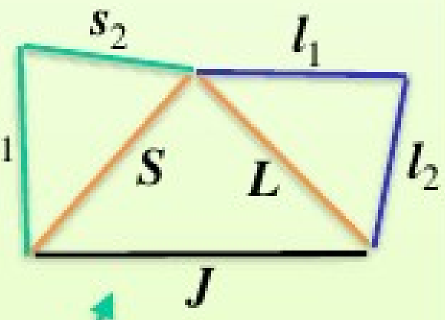


$[SL]_J$

Cambio de base SL —> jj



$$= \sum_{S,L} \langle s_1 l_1(j_1), s_2 l_2(j_2); jm \,|\, s_1 s_2(S) l_1 l_2(L); jm \rangle$$



$|\, s_1 l_1(j_1), s_2 l_2(j_2); jm \rangle$

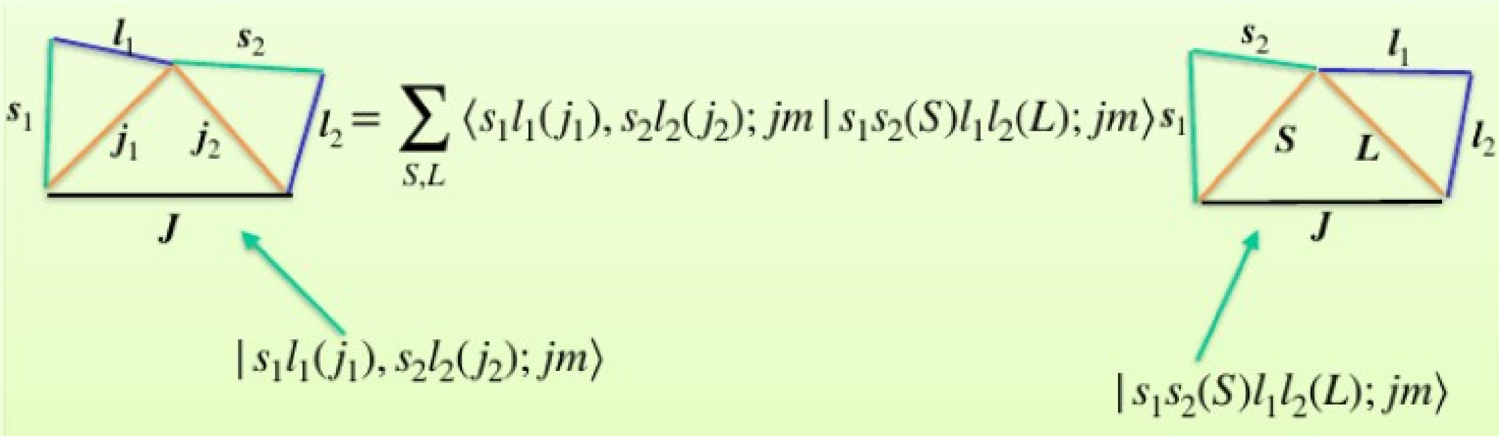
$|\, s_1 s_2(S) l_1 l_2(L); jm \rangle$

$$|\, s_1 l_1(j_1), s_2 l_2(j_2); jm \rangle = \sum_{S,L} \hat{j}_1 \hat{j}_2 \hat{S} \hat{L} \begin{bmatrix} s_1 & s_2 & S \\ l_1 & l_2 & L \\ j_1 & j_2 & j \end{bmatrix} |\, s_1 s_2(S) l_1 l_2(L); jm \rangle$$

Volveremos sobre estos acoples al definir la función de onda de dos nucleones

Coeficiente de Wigner 9j

Cambio de base dos partículas



Estado fundamental

$$\Psi^{00}_{aa}(\boldsymbol{r}_1, \boldsymbol{r}_2) \quad = \quad R_a(r_1) R_a(r_2) \sum_S \hat{j}_a^2 \hat{S}^2 \begin{Bmatrix} \frac{1}{2} & l_a & j_a \\ \frac{1}{2} & l_a & j_a \\ S & S & 0 \end{Bmatrix} [\chi(1)\chi(2))]_{SM_S} \, [Y_{l_a}(\hat{r}_1) Y_{l_a}(\hat{r}_2)]_{SM_S}$$

Singlete S=0

$|\bar{r}_1| = |\bar{r}_2| = r$

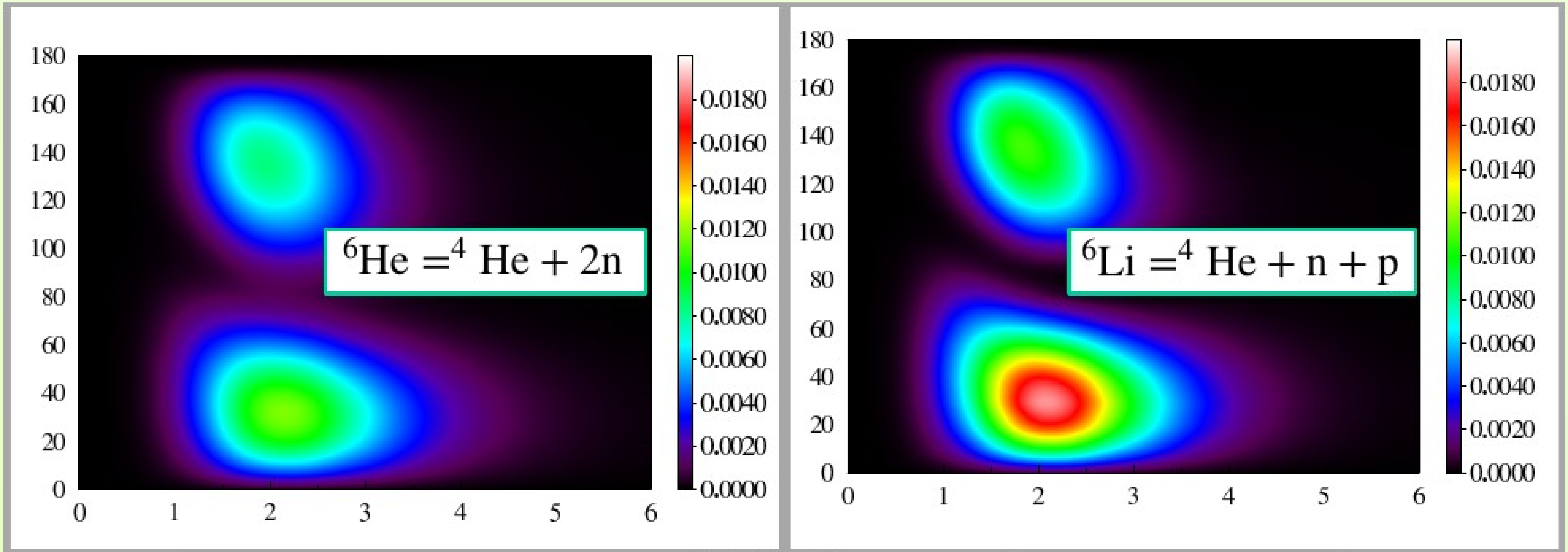
$$\begin{Bmatrix} \frac{1}{2} & l_p & j_p \\ \frac{1}{2} & l_p & j_p \\ 0 & 0 & 0 \end{Bmatrix} = \frac{1}{\hat{\frac{1}{2}} \hat{l}_p \hat{j}_p} = \frac{\sqrt{2}}{\hat{l}_p \hat{j}_p} \qquad [Y_l(\hat{r}_1) Y_l(\hat{r}_2)]_{00} \quad = \quad (-)^{l+1} \frac{\hat{l}}{4\pi} P_l(\cos\theta_{12})$$

$$\Psi^{00}_{aa}(\boldsymbol{r}_1, \boldsymbol{r}_2)\Big|_{S=0} \quad = \quad \frac{(-)^{l_a}}{4\pi} \, \frac{\hat{j}_a}{\sqrt{2}} \, R_a^2(r) \, P_{l_a}(\cos\theta_{12}) \, [\chi(1)\chi(2))]_{00}$$

Densidad del singlete

$$\Psi^{00}_{aa}(\boldsymbol{r}_1, \boldsymbol{r}_2)\Big|_{S=0} \quad = \quad \frac{(-)^{l_a}}{4\pi}\,\frac{\hat{j}_a}{\sqrt{2}}\,R^2_a(r)\,P_{l_a}(\cos\theta_{12})\,[\chi(1)\chi(2)]_{00}$$

$$|\bar{r}_1| = |\bar{r}_2| = r$$



$\rho_{^6He}(R, \theta_{12})(0^+)$

Crédito: Yannen

$\rho_{^6Li}(R, \theta_{12})(1^+)$

Fin