

Trabajo Práctico Nº1: Sistemas de Ecuaciones Lineales - Soluciones

Matriz de coeficientes:

$$\begin{bmatrix} 1 & -1 & 0 & 0 & -1 & 0 \\ -1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 & -1 \\ R_1 & 0 & -R_3 & 0 & R_5 & 0 \\ 0 & R_2 & 0 & 0 & -R_5 & R_6 \\ 0 & 0 & R_3 & R_4 & 0 & -R_6 \\ R_1 & R_2 & 0 & R_4 & 0 & 0 \\ R_1 & R_2 & -R_3 & 0 & 0 & R_6 \\ R_1 & 0 & 0 & R_4 & R_5 & -R_6 \\ 0 & R_2 & R_3 & R_4 & -R_5 & 0 \end{bmatrix} \quad (1)$$

Matriz de coeficientes ampliada:

$$\left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 \\ R_1 & 0 & -R_3 & 0 & R_5 & 0 & E_1 - E_3 \\ 0 & R_2 & 0 & 0 & -R_5 & R_6 & E_2 \\ 0 & 0 & R_3 & R_4 & 0 & -R_6 & E_3 + E_4 \\ R_1 & R_2 & 0 & R_4 & 0 & 0 & E_1 + E_2 + E_4 \\ R_1 & R_2 & -R_3 & 0 & 0 & R_6 & E_1 + E_2 - E_3 \\ R_1 & 0 & 0 & R_4 & R_5 & -R_6 & E_1 + E_4 \\ 0 & R_2 & R_3 & R_4 & -R_5 & 0 & E_2 + E_3 + E_4 \end{array} \right] \quad (2)$$

Reducción de la matriz de coeficientes ampliada a su forma escalonada: para el caso que $E_1 = E_2 = E_3 = R_3 = 0$.

$$\left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 \\ R_1 & 0 & 0 & 0 & R_5 & 0 & 0 \\ 0 & R_2 & 0 & 0 & -R_5 & R_6 & 0 \\ 0 & 0 & 0 & R_4 & 0 & -R_6 & E_4 \\ R_1 & R_2 & 0 & R_4 & 0 & 0 & E_4 \\ R_1 & R_2 & 0 & 0 & 0 & R_6 & 0 \\ R_1 & 0 & 0 & R_4 & R_5 & -R_6 & E_4 \\ 0 & R_2 & 0 & R_4 & -R_5 & 0 & E_4 \end{array} \right] \quad (3)$$

Operación: fila 11 = fila 6 - fila 11

$$\left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 \\ R_1 & 0 & 0 & 0 & R_5 & 0 & 0 \\ 0 & R_2 & 0 & 0 & -R_5 & R_6 & 0 \\ 0 & 0 & 0 & R_4 & 0 & -R_6 & E_4 \\ R_1 & R_2 & 0 & R_4 & 0 & 0 & E_4 \\ R_1 & R_2 & 0 & 0 & 0 & R_6 & 0 \\ R_1 & 0 & 0 & R_4 & R_5 & -R_6 & E_4 \\ 0 & 0 & 0 & -R_4 & 0 & R_6 & -E_4 \end{array} \right] \quad (4)$$

Operación: fila 11 = fila 7 + fila 11

$$\left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 \\ R_1 & 0 & 0 & 0 & R_5 & 0 & 0 \\ 0 & R_2 & 0 & 0 & -R_5 & R_6 & 0 \\ 0 & 0 & 0 & R_4 & 0 & -R_6 & E_4 \\ R_1 & R_2 & 0 & R_4 & 0 & 0 & E_4 \\ R_1 & R_2 & 0 & 0 & 0 & R_6 & 0 \\ R_1 & 0 & 0 & R_4 & R_5 & -R_6 & E_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \quad (5)$$

Operación: fila 10 = fila 5 - fila 10

$$\left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 \\ R_1 & 0 & 0 & 0 & R_5 & 0 & 0 \\ 0 & R_2 & 0 & 0 & -R_5 & R_6 & 0 \\ 0 & 0 & 0 & R_4 & 0 & -R_6 & E_4 \\ R_1 & R_2 & 0 & R_4 & 0 & 0 & E_4 \\ R_1 & R_2 & 0 & 0 & 0 & R_6 & 0 \\ 0 & 0 & 0 & -R_4 & 0 & R_6 & -E_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \quad (6)$$

Operación: fila 10 = fila 7 + fila 10

$$\left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 \\ R_1 & 0 & 0 & 0 & R_5 & 0 & 0 \\ 0 & R_2 & 0 & 0 & -R_5 & R_6 & 0 \\ 0 & 0 & 0 & R_4 & 0 & -R_6 & E_4 \\ R_1 & R_2 & 0 & R_4 & 0 & 0 & E_4 \\ R_1 & R_2 & 0 & 0 & 0 & R_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \quad (7)$$

Operación: fila 9 = fila 8 - fila 9

$$\left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 \\ R_1 & 0 & 0 & 0 & R_5 & 0 & 0 \\ 0 & R_2 & 0 & 0 & -R_5 & R_6 & 0 \\ 0 & 0 & 0 & R_4 & 0 & -R_6 & E_4 \\ R_1 & R_2 & 0 & R_4 & 0 & 0 & E_4 \\ 0 & 0 & 0 & R_4 & 0 & -R_6 & E_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \quad (8)$$

Operación: fila 9 = fila 7 - fila 9

$$\left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 \\ R_1 & 0 & 0 & 0 & R_5 & 0 & 0 \\ 0 & R_2 & 0 & 0 & -R_5 & R_6 & 0 \\ 0 & 0 & 0 & R_4 & 0 & -R_6 & E_4 \\ R_1 & R_2 & 0 & R_4 & 0 & 0 & E_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \quad (9)$$

Operación: fila 8 = fila 5 - fila 8

$$\left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 \\ R_1 & 0 & 0 & 0 & R_5 & 0 & 0 \\ 0 & R_2 & 0 & 0 & -R_5 & R_6 & 0 \\ 0 & 0 & 0 & R_4 & 0 & -R_6 & E_4 \\ 0 & -R_2 & 0 & -R_4 & R_5 & 0 & -E_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \quad (10)$$

Operación: fila 8 = fila 6 + fila 8

$$\left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 \\ R_1 & 0 & 0 & 0 & R_5 & 0 & 0 \\ 0 & R_2 & 0 & 0 & -R_5 & R_6 & 0 \\ 0 & 0 & 0 & R_4 & 0 & -R_6 & E_4 \\ 0 & 0 & 0 & -R_4 & 0 & R_6 & -E_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \quad (11)$$

Operación: fila 8 = fila 7 + fila 8

$$\left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 \\ R_1 & 0 & 0 & 0 & R_5 & 0 & 0 \\ 0 & R_2 & 0 & 0 & -R_5 & R_6 & 0 \\ 0 & 0 & 0 & R_4 & 0 & -R_6 & E_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \quad (12)$$

Operación: fila 2 = fila 1 + fila 2

$$\left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 \\ R_1 & 0 & 0 & 0 & R_5 & 0 & 0 \\ 0 & R_2 & 0 & 0 & -R_5 & R_6 & 0 \\ 0 & 0 & 0 & R_4 & 0 & -R_6 & E_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \quad (13)$$

Operación: fila 1 = fila 1 + fila 4

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & -1 & -1 & -1 & 0 \\ 0 & -1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 \\ R_1 & 0 & 0 & 0 & R_5 & 0 & 0 \\ 0 & R_2 & 0 & 0 & -R_5 & R_6 & 0 \\ 0 & 0 & 0 & R_4 & 0 & -R_6 & E_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \quad (14)$$

Operación: fila 2 = fila 2 + fila 3

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & -1 & -1 & -1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 \\ R_1 & 0 & 0 & 0 & R_5 & 0 & 0 \\ 0 & R_2 & 0 & 0 & -R_5 & R_6 & 0 \\ 0 & 0 & 0 & R_4 & 0 & -R_6 & E_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \quad (15)$$

Operación: fila 4 = fila 2 + fila 4

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & -1 & -1 & -1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ R_1 & 0 & 0 & 0 & R_5 & 0 & 0 \\ 0 & R_2 & 0 & 0 & -R_5 & R_6 & 0 \\ 0 & 0 & 0 & R_4 & 0 & -R_6 & E_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \quad (16)$$

Operación: fila 5 = fila 5 - R_1 fila 1

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & -1 & -1 & -1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & R_1 & R_1 + R_5 & R_1 & 0 \\ 0 & R_2 & 0 & 0 & -R_5 & R_6 & 0 \\ 0 & 0 & 0 & R_4 & 0 & -R_6 & E_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \quad (17)$$

Operación: fila 6 = R_2 fila 2 + fila 6

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & -1 & -1 & -1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & R_1 & R_1 + R_5 & R_1 & 0 \\ 0 & 0 & 0 & R_2 & R_2 - R_5 & R_2 + R_6 & 0 \\ 0 & 0 & 0 & R_4 & 0 & -R_6 & E_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \quad (18)$$

Operación: fila 6 = R_2 fila 5 - R_1 fila 6

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & -1 & -1 & -1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & R_1 & R_1 + R_5 & R_1 & 0 \\ 0 & 0 & 0 & 0 & R_5(R_1 + R_2) & -R_1R_6 & 0 \\ 0 & 0 & 0 & R_4 & 0 & -R_6 & E_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \quad (19)$$

Operación: fila 7 = R_4 fila 5 - R_1 fila 7

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & -1 & -1 & -1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & R_1 & R_1 + R_5 & R_1 & 0 \\ 0 & 0 & 0 & 0 & R_5(R_1 + R_2) & -R_1R_6 & 0 \\ 0 & 0 & 0 & 0 & R_4R_1 + R_4R_5 & R_4R_1 + R_1R_6 & -R_1E_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \quad (20)$$

Operación: fila 6 = fila 6 / ($R_5(R_1 + R_2)$)

Operación: fila 7 = fila 7/($R_4R_1 + R_4R_5$)

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & -1 & -1 & -1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & R_1 & R_1 + R_5 & R_1 & 0 \\ 0 & 0 & 0 & 0 & 1 & f_1 & 0 \\ 0 & 0 & 0 & 0 & 1 & f_2 & -f_3E_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \quad (21)$$

donde

$$f_1 = -\frac{R_1R_6}{R_5(R_1 + R_2)} \quad (22)$$

$$f_2 = \frac{R_1(R_4 + R_6)}{R_4(R_1 + R_5)} \quad (23)$$

$$f_3 = \frac{R_1}{R_4(R_1 + R_5)} \quad (24)$$

Operación: fila 7 = fila 6 - fila 7

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & -1 & -1 & -1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & R_1 & R_1 + R_5 & R_1 & 0 \\ 0 & 0 & 0 & 0 & 1 & f_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & f_1 - f_2 & f_3E_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \quad (25)$$

Operación: fila 2 = - fila 2

Operación: fila 5 = fila 5/ R_1

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & -1 & -1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & f_4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & f_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & f_1 - f_2 & f_3E_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \quad (26)$$

donde

$$f_4 = \frac{R_1 + R_5}{R_1} \quad (27)$$

Operación: se intercambia la fila 4 sucesivamente con la inmediatamente inferior hasta la fila 7

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & -1 & -1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & f_4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & f_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & f_1 - f_2 & f_3 E_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \quad (28)$$

Forma escalonada con $E_1 = E_2 = E_3 = R_3 = 0$:

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & -1 & -1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & f_4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & f_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & f_1 - f_2 & f_3 E_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \quad (29)$$

con

$$f_1 = -\frac{R_1 R_6}{R_5(R_1 + R_2)} \quad (30)$$

$$f_2 = \frac{R_1(R_4 + R_6)}{R_4(R_1 + R_5)} \quad (31)$$

$$f_3 = \frac{R_1}{R_4(R_1 + R_5)} \quad (32)$$

$$f_4 = \frac{R_1 + R_5}{R_1} \quad (33)$$

$$(34)$$

Rango de la matriz A de coeficientes: 6 (seis)

Rango de la matriz ampliada: 6 (seis)

Número de variable libres: número incógnitas - Rank(A) = 0 (cero)

Sistema de ecuaciones lineales a partir de la matriz escalonada (63)

$$I_1 - I_4 - I_5 - I_6 = 0 \quad (35)$$

$$I_2 - I_4 - I_6 = 0 \quad (36)$$

$$I_3 + I_5 + I_6 = 0 \quad (37)$$

$$I_4 + f_4 I_5 + I_6 = 0 \quad (38)$$

$$I_5 + f_1 I_6 = 0 \quad (39)$$

$$(f_1 - f_2) I_6 = f_3 E_4 \quad (40)$$

Corrientes para: $E_4 = 12 \text{ V}$, $R_1 = R_2 = R_4 = 1 \Omega$, $R_5 = R_6 = 2 \Omega$

Los valores de las constantes resultan,

$$f_1 = -\frac{1}{2} \quad (41)$$

$$f_2 = 1 \quad (42)$$

$$f_3 = \frac{1}{3} \quad (43)$$

$$f_4 = 3 \quad (44)$$

$$f_1 - f_2 = -\frac{3}{2} \quad (45)$$

Luego,

$$I_1 - I_4 - I_5 - I_6 = 0 \quad (46)$$

$$I_2 - I_4 - I_6 = 0 \quad (47)$$

$$I_3 + I_5 + I_6 = 0 \quad (48)$$

$$I_4 + 3 I_5 + I_6 = 0 \quad (49)$$

$$I_5 - \frac{1}{2} I_6 = 0 \quad (50)$$

$$-\frac{3}{2} I_6 = \frac{1}{3} 12 = 4 \quad (51)$$

Luego, por backsubstitution obtenemos,

$$I_6 = -\frac{8}{3} A \quad (52)$$

$$I_5 = -\frac{4}{3} A \quad (53)$$

$$I_4 = \frac{20}{3} A \quad (54)$$

$$I_3 = 4 A \quad (55)$$

$$I_2 = 4 A \quad (56)$$

$$I_1 = \frac{8}{3} A \quad (57)$$

Reducción del sistema (46)-(51) a su forma escalonada reducida

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & -1 & -1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & -3/2 & 4 \end{array} \right] \quad (58)$$

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & -1 & -1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -8/3 \end{array} \right] \quad (59)$$

Trabajo para hacer ceros en la sexta columna,

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & -1 & -1 & 0 & -8/3 \\ 0 & 1 & 0 & -1 & 0 & 0 & -8/3 \\ 0 & 0 & 1 & 0 & 1 & 0 & 8/3 \\ 0 & 0 & 0 & 1 & 3 & 0 & 8/3 \\ 0 & 0 & 0 & 0 & 2 & 0 & -8/3 \\ 0 & 0 & 0 & 0 & 0 & 1 & -8/3 \end{array} \right] \quad (60)$$

Divido por dos la quinta fila

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & -1 & -1 & 0 & -8/3 \\ 0 & 1 & 0 & -1 & 0 & 0 & -8/3 \\ 0 & 0 & 1 & 0 & 1 & 0 & 8/3 \\ 0 & 0 & 0 & 1 & 3 & 0 & 8/3 \\ 0 & 0 & 0 & 0 & 1 & 0 & -4/3 \\ 0 & 0 & 0 & 0 & 0 & 1 & -8/3 \end{array} \right] \quad (61)$$

Trabajo para hacer ceros en la quinta columna,

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & -1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -1 & 0 & 0 & -8/3 \\ 0 & 0 & 1 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 1 & 0 & 0 & 20/3 \\ 0 & 0 & 0 & 0 & 1 & 0 & -4/3 \\ 0 & 0 & 0 & 0 & 0 & 1 & -8/3 \end{array} \right] \quad (62)$$

Trabajo para hacer ceros en la cuarta columna,

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & 0 & 0 & 0 & 8/3 \\ 0 & 1 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 1 & 0 & 0 & 20/3 \\ 0 & 0 & 0 & 0 & 1 & 0 & -4/3 \\ 0 & 0 & 0 & 0 & 0 & 1 & -8/3 \end{array} \right] \quad (63)$$

Luego,

$$I_1 = 8/3 A \quad (64)$$

$$I_2 = 4 A \quad (65)$$

$$I_3 = 4 A \quad (66)$$

$$I_4 = 20/3 A \quad (67)$$

$$I_5 = -4/3 A \quad (68)$$

$$I_6 = -8/3 A \quad (69)$$